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STATE UNIVERSITY OF NEW YORK AT BUFFALO

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FINAL TECHNICAL REPORT

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14. ABSTRACT This report summarizes results and findings of the project including: (1) addressing the fundamental problem of determining optimal transmission power sequence for cooperative hybrid automatic-repeat-request (H-ARQ) relaying protocol in wireless networks; and (2) designing and optimizing cooperative communication protocols by exploring all possible variations in time and power domains for wireless networks.					
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1 Executive Summary

1.1 Overview and Main Results

This report describes work performed from 25 July 2010 to 19 October 2013 under AFRL grant number FA8750-11-1-0015 entitled “Studies on Optimized Assured Cooperative Communications”. We cover two important tasks: (1) determine optimal transmission power sequence for cooperative hybrid automatic-repeat-request (H-ARQ) relaying protocol in wireless networks; and (2) design and optimize cooperative communication protocols by exploring all possible variations in time and power domains. Some main results are summarized in the following:

- We first addressed the fundamental problem of assigning optimal transmission power sequence for cooperative hybrid automatic-repeat-request (H-ARQ) relaying protocol over quasi-static Rayleigh fading channels. A closed-form expression of the average total transmission power was obtained first for the cooperative H-ARQ relaying protocol, in which the source may use different transmission power level in different (re)transmission rounds. We determined the optimal power sequence by minimizing the average total transmission power. However, the closed-form expression of the average total power consumption of the cooperative H-ARQ protocol is complicated in general, so we developed a simple approximation of the average total transmission power that is tight at high SNR. Based on the asymptotically tight approximation, we were able to identify the sequence of power values that minimizes the average total power consumption of the cooperative H-ARQ protocol for any given targeted outage probability. In particular, we derived a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. When the maximum number of (re)transmissions allowed in the protocol is $L = 2$, we have a closed-form result for the optimal transmission power sequence. The optimal power assignment solution reveals that conventional equal power assignment is not optimal in general. For example, for targeted outage probability of 10^{-4} with a maximum of two (re)transmissions, the average total transmission power with the optimal power assignment is 3 dB lower than the equal power assignment. The difference in average total power cost grows further when the number of allowable (re)transmissions increases (for example, 6 dB gain with a cap of 4 (re)transmissions) or the targeted outage probability decreases (11 dB gain with outage probability 10^{-5} and (re)transmission capped at 4).
- Cooperative communication has emerged as a new wireless network communication concept, in which parameter optimization such as power budget and time allocation plays an important role in cooperative relaying protocol designs. While most existing works on cooperative relaying protocol designs considered equal-time allocation scenario, i.e. equal time duration is assigned to each source and each relay, in this project we designed and optimized cooperative communication protocols by exploring all possible variations in time and power domains. First, we considered an ideal cooperative relaying protocol where the system can use arbitrary re-encoding methods at the relay and adjust time allocation arbitrarily. We obtained an optimum strategy of power and time allocations to minimize the outage probability of the ideal cooperative protocol. Specifically, for any given time allocation, we were able to determine the corresponding optimum power allocation analytically with a closed-form expression. We also showed that in order to minimize the outage probability of the protocol, one should always allocate more energy and time to the source than the relay. Second, with more realistic consideration, we designed a practical cooperative relaying protocol based on linear mapping, i.e. using linear mapping as the re-encoding method at the relay and considering integer time slots in the two phases. The theoretical results from the ideal cooperative protocol served as guideline and benchmark in the practical cooperative protocol design. We also developed an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Extensive numerical and simulation studies illustrate our theoretical developments and show that the performance of the proposed cooperative relaying protocol based on the optimum linear mapping is close to the performance benchmark of the ideal cooperative protocol.

The introduction to the project and an overview of each research task is given in Section 2. Then in Section 3 we present specifically models, assumptions, methods, and main results for each task. Section 4 contains the conclusions and bibliography is included at the end.

1.2 List of People Involved

There are four faculty members involved in the project, which are listed in the following:

- Prof. Weifeng Su; University at Buffalo, State University of New York (SUNY); PI
- Prof. Stella Batalama; University at Buffalo, State University of New York (SUNY); Collaborator
- Prof. Dimitris Pados; University at Buffalo, State University of New York (SUNY); Collaborator
- Prof. Tommaso Melodia; University at Buffalo, State University of New York (SUNY); Collaborator

Additionally, the project supported two Ph.D students, working in the project toward their Ph.D degrees.

1.3 List of Publications

The project has resulted in the following peer-reviewed journal articles and conference proceeding papers.

- S. Lee, W. Su, D. A. Pados, and J. D. Matyjas, "The optimal power assignment for cooperative hybrid-ARQ relaying protocol," in Proceedings of IEEE Global Telecommunications Conference (GLOBE-COM), pp.1-6, Houston, TX, Dec. 5-9, 2011.
- W. Su, S. Lee, D. A. Pados, and J. D. Matyjas, "Optimal power assignment for minimizing the average total transmission power in hybrid-ARQ Rayleigh fading links," IEEE Transactions on Communications, vol. 59, no. 7, pp.1867-1877, July 2011.
- W. Su, J. D. Matyjas, and S. N. Batalama, "Active cooperation between primary users and cognitive radio users in heterogeneous ad-hoc networks," IEEE Transactions on Signal Processing, vol. 60, no. 4, pp.1796-1805, April 2012.
- S. Lee, W. Su, D. A. Pados, and J. D. Matyjas, "Optimal power assignment to minimize the average delay in hybrid-ARQ protocols," in Proceedings of IEEE International Conference on Communications (ICC), Ottawa, Canada, June 10-15, 2012.
- W. Su, J. D. Matyjas, M. J. Gans, and S. N. Batalama, "On the capacity of airborne MIMO communications," in Proceedings of IEEE Global Telecommunications Conference (GLOBECOM), Anaheim, CA, December 3-7, 2012.
- W. Su, J. D. Matyjas, M. J. Gans, and S. N. Batalama, "Maximum achievable capacity in airborne MIMO communications with arbitrary alignments of linear transceiver antenna arrays," IEEE Transactions on Wireless Communications, vol. 12, no. 11, pp.5584-5593, November 2013.
- Z. Mo, W. Su, S. N. Batalama, and J. D. Matyjas, "Cooperative communication protocol designs based on optimum power and time allocation," submitted to IEEE Transactions on Wireless Communications, August 2013.
- Z. Mo, W. Su, S. N. Batalama, and J. D. Matyjas, "Optimum power and time allocation for DF cooperative relaying protocol," submitting to IEEE International Conference on Communications (ICC'14), 2013.
- Z. Mo, W. Su, S. N. Batalama, and J. D. Matyjas, "Linear-mapping based cooperative relaying protocol design with optimum power and time allocation," submitting to IEEE International Conference on Communications (ICC'14), 2013.

2 Introduction to the Project

2.1 Overview of Cooperative ARQ Relaying Protocols in Wireless Networks

Cooperative wireless networks can substantially increase network reliability as each user's information may be jointly delivered to its destination with the assistance of cooperative users/nodes in the networks [1]–[8]. On the other hand, automatic-repeat-request (ARQ) protocols have been long-time used to enable reliable data packet transmissions in data link control [9]–[12], in which a receiver requests retransmission when a packet is not correctly received. In basic ARQ protocols, a receiver decodes an information packet based only on the received signal in each (re)transmission round [9, 10], while in more advanced ARQ protocols, a receiver may decode an information packet by combining received signals from all previous (re)transmission rounds, resulting in so-called hybrid ARQ (H-ARQ) protocols [11, 12].

It is a natural idea to exploit H-ARQ protocols in conjunction with the cooperative communication concept to jointly enhance link connectivity and network reliability. When a source sends a wireless signal to an intended destination, nearby users may also receive the signal. Thus, if the destination requests retransmission, the nearby users may also assist forwarding the signal alongside the source's retransmission. The destination can combine all received signals from source and relays and jointly decode the signals to improve detection performance. Some recent works have studied H-ARQ protocols in the context of cooperative relay networks [13]–[16]. In particular, [13] was among the first such studies to present a general framework of cooperative ARQ relay networks. It was shown that cooperative ARQ relay networks have great advantages in terms of throughput, delay, and energy consumption compared to conventional multihop ARQ networks in which point-to-point ARQ links are concatenated to form network routes. In [14, 15], information-theoretic analysis was developed and upper bounds for the diversity order of a decode-and-forward (DF) cooperative ARQ relay scheme were characterized for both slow and fast fading channels as a means to study the diversity-multiplexing-delay tradeoff. Recently, a closed-form asymptotically tight approximation of the outage probability for the DF cooperative ARQ relay scheme was developed under fast fading conditions [16]. It was shown that the cooperative ARQ scheme achieves diversity order equal to $2L - 1$ while the diversity order of the direct ARQ transmission scheme is only L , where L is the maximum number of (re)transmission rounds.

In wireless links formed by wireless devices with limited power resources, power efficiency is a key research matter in the optimization of ARQ retransmission protocols [17]–[21]. In both [17] and [18], the power efficiency of ARQ protocols was examined under the assumption of the same transmission power level in each (re)transmission round. In [19], by assuming that partial channel state information (CSI) is available, optimal transmission power in each (re)transmission round was determined for an H-ARQ protocol by a linear programming method that selects a power value from a set of discrete power levels. In [20], without assuming CSI available at the transmitter side, an optimal power transmission strategy was identified for a basic ARQ protocol where the receiver decodes based only on the received signal in each (re)transmission round. It was assumed that the channel changes independently in each round. In [21], by a recursive calculation, an optimal power assignment sequence for an H-ARQ protocol was determined in quasi-static Rayleigh fading, in which the channel does not change during (re)transmissions of the same information packet.

Note that, while the average total power consumption needed in the delivery of each information packet has been well understood for the conventional non-cooperative H-ARQ protocols [19]–[21], the study of cooperative H-ARQ counterpart has been proven very challenging with no available results until this present work. In this project, we try to determine the optimal power sequence assignment that minimizes the average total power consumption for the cooperative H-ARQ relaying protocol in which each relay forwards Alamouti-based retransmission signals. We consider a quasi-static Rayleigh fading environment by assuming that the channels do not change during (re)transmissions of the same information packet and they may change independently when the protocol transmits a new information packet. We develop a new analytical approach and obtain a closed-form expression of the average total transmission power. The closed-form result is valid for any maximum number of (re)transmission rounds L allowed in the protocol in which the source may use varying transmission power level per round. The closed-form expression may serve, therefore, as the basis to optimize power allocation for the cooperative H-ARQ relaying protocol. However, the closed-form expression is rather complicated

for large L , so we develop a simple approximation of the average total transmission power which is tight at high SNR. Based on the asymptotically tight approximation, we are able to identify the sequence of power values that minimizes the average total power consumption of the cooperative H-ARQ relaying protocol for any given targeted outage probability. In particular, we derive a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. When the maximum number of (re)transmissions allowed in the protocol is $L = 2$, we have a closed-form result for the optimal transmission power sequence. The optimal power assignment solution reveals that conventional equal power assignment scheme which uses the same transmission power in all (re)transmission rounds is not optimal. As an example, for a targeted outage probability of 10^{-4} and the maximum number of (re)transmissions $L = 3$, the average total transmission power based on the optimal power assignment is 6 dB less than that of using the common equal power scheme. We also observe that the larger the maximum number of (re)transmissions allowed in the cooperative H-ARQ protocol or the lower the required outage probabilities, the more power savings the optimal power assignment strategy offers. Extensive simulation and numerical results are provided to illustrate and validate our theoretical development.

2.2 Overview of Cooperative Communication Protocol Designs Based on Optimum Power and Time Allocation

Cooperative communication has received significant attention recently as an emerging communication concept for wireless networks [24, 25]. Due to the broadcast nature of wireless transmissions, cooperative communications enables neighboring network nodes to share resources and cooperate to send information to an intended node. Distributed transmissions from source and relay nodes provide spatial diversity (as well as multiplexing gain in some designs) for information detection at a destination node. Cooperative communications can significantly improve system performance and robustness of wireless networks especially in severe fading environment. Cooperative relaying techniques have been considered in some latest communication standards, for example, in IEEE 802.16j WiMAX standard [26] and 3GPP's Long Term Evolution (LTE)-Advanced standard [27].

The idea of cooperative communications can be traced back to 1970's [28] and information-theoretic studies have been extensively carried out since then (see [29, 30, 31, 32, 33] and the references therein). In recent years, besides information-theoretic studies, many efforts have been shifted to design practical cooperative communication protocols for wireless networks with specific system constraints and quality-of-service (QoS) requirements. Various cooperative relaying protocols such as decode-and-forward (DF) relaying protocol and amplify-and-forward (AF) relaying protocol were proposed for wireless networks (see, for examples, [34, 35, 36, 37]) and substantial performance gains of such relaying protocols have been demonstrated compared to conventional non-cooperative transmission approach. Cooperative relaying protocols have been generalized to multi-relay networks with either parallel or sequential multiple relays to further improve network performance with price of higher complexity and more overhead [34, 38]. Cooperative communication protocols have also been integrated into conventional QoS control mechanisms such as automatic-repeat-request (ARQ) protocols to enhance reliability and robustness of wireless networks [39, 40]. Cooperative relaying techniques have been applied to multimedia wireless broadcast and multicast applications with substantial data rate increase and power saving [41, 42]. More thorough discussions on basic theories, protocols, and applications for cooperative communications can be found in [24, 25].

Resource allocation in cooperative communication protocols such as power budget and time allocation to source and relays plays an important role in the overall performance of the protocols. Note that most existing works on cooperative relaying protocol designs considered equal-time allocation scenario, i.e. equal time duration is assigned to each source and each relay [34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45]. For example, with equal-time allocation, in [43] source and relay power allocation was optimized such that the outage probability of the cooperative relaying protocol is minimized. In [44], also with equal-time allocation, optimum power allocation was determined in which symbol error rate (SER) performances of both DF and AF relaying protocols were optimized respectively. However, there are rather limited studies on *non-equal* time allocation scenario and further on joint optimization of power allocation and time allocation in cooperative relaying pro-

tol designs. In [46], by assuming that instantaneous channel state information (CSI) of source-destination, source-relay and relay-destination links are available, joint power and time allocation were optimized in order to minimize the outage probability of the cooperative communication system. In [47], it was shown that the problem of minimizing the outage probability with respect to joint power and time allocation at source and relays is a multi-variable convex optimization problem for nonorthogonal cooperative communication systems where the source and the relay are allowed to transmit simultaneously. However, numerical method was considered in [47] to solve the convex optimization problem without analytical solutions. It is hard to obtain insight understanding of the cooperative relaying protocols based on the numerical results. We note that in both [46] and [47], independent codebooks were used for source and relays in the cooperative relaying protocol designs.

In this project, we intend to design and optimize cooperative communication protocols by exploring all possible variations in time and power domains. We assume that the channels are known at the receiver side, but not at the transmitter side. We consider an orthogonal cooperative communication scenario, in which the source transmits signals in Phase I and the relay decodes the signals and forwards them to the destination in Phase II. First, we analyze an ideal cooperative communication protocol where the system can use arbitrary re-encoding methods at the relay and adjust time allocation arbitrarily between Phases I and II. We are able to obtain an optimum strategy of power and time allocations to minimize the outage probability of the ideal cooperative protocol. Specifically, for any given time allocation, we are able to determine the corresponding optimum power allocation at the source and relay analytically with a closed-form expression. We also show theoretically that in order to minimize the outage probability of the protocol, one should always allocate more energy and time to Phase I than that to Phase II in the protocol.

Note that in the ideal cooperative protocol in which there is no constraint on the re-encoding methods and time allocation, it may not be easy or feasible to implement it in practical systems. So, next we would like to propose a practical cooperative communication protocol design based on linear mapping. The theoretical results from the ideal cooperative protocol will serve as guideline and benchmark in the practical cooperative protocol design. More specifically, we design a cooperative communication protocol by considering linear mapping as the re-encoding method at the relay, where the protocol uses integer time slots in Phases I and II. It is much easier to implement the linear mapping forwarding method with the time allocation of integer time slots in the two phases in practical systems. We also develop an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Interestingly, simulation results show that the performance of the proposed cooperative protocol based on the optimum linear mapping is close to the performance benchmark of the ideal cooperative protocol.

3 Models, Assumptions, Methods and Procedures

3.1 Cooperative Hybrid-ARQ Relaying in Wireless Networks: Average Power Consumption and Optimal Power Assignment

In this subsection, we determine optimal transmission power sequence for cooperative hybrid automatic-repeat-request (H-ARQ) relaying protocol in wireless networks. First, we describe briefly the cooperative H-ARQ relaying protocol. Second, we derive the average total transmission power expended in the delivery of each information packet. Third, we develop an approximation of the average total transmission power which is tight at high SNR. Fourth, we determine an optimal power assignment strategy for the cooperative H-ARQ relaying protocol based on the approximation of the average total transmission power developed in the previous section. Finally, numerical and simulation studies are carried out to compare the performance of the optimal and equal power assignment strategies.

3.1.1 System Model

For simplicity in presentation, we consider a cooperative H-ARQ relaying model with one source, one relay and one destination, as illustrated in Fig. 1. We assume that L is the maximum number of (re)transmission rounds allowed in the protocol. The cooperative H-ARQ relaying scheme operates as follows. First, the source

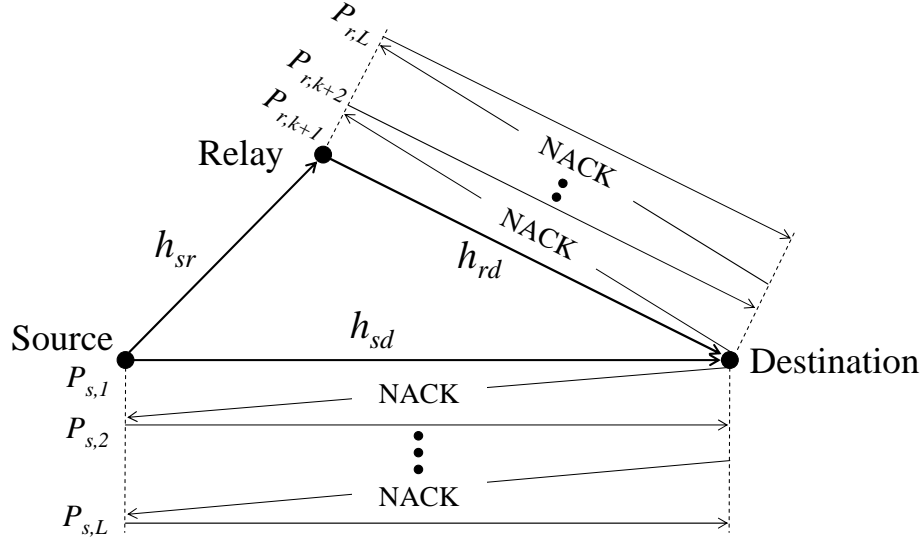


Figure 1: Illustration of a cooperative H-ARQ relaying protocol with source (re)transmission power $P_{s,m}$ ($1 \leq m \leq L$) and relay power $P_{r,n}$ ($k+1 \leq n \leq L$) when the relay decodes correctly at the k -th round and starts cooperating at round $(k+1)$.

broadcasts an information packet to the destination and the relay. The destination sends a single bit of acknowledgement (ACK) or negative-acknowledgement (NACK) indicating success or failure of receiving the packet, respectively, to both the source and the relay. The ACK/NACK feedback is assumed to be detected error-free at the source and the relay. If ACK is received by the source or retransmission reaches the maximum number of rounds L , the source begins transmission of a new information packet. If NACK is received by the source and the maximum number of rounds L is not reached, the source retransmits the packet at a potentially different transmission power. If the relay decodes successfully ahead of the destination, the relay starts cooperating with the source by forwarding the packet to the destination by using a space-time transmission scheme [14], for example the Alamouti scheme [23]. The destination combines the signals from the source and the signals from the relay and jointly decodes the information packet. If the destination still cannot decode an information packet after L (re)transmission rounds, an outage event is declared which means that the signal-to-noise (SNR) of the combined received signals is below a required SNR. The probability of an outage event is referred to as the outage probability.

The cooperative H-ARQ relaying scheme can be modeled as follows. The received signal $y_{r,m}$ at the relay at the m -th ($1 \leq m \leq L$) H-ARQ (re)transmission round can be modeled as

$$y_{r,m} = \sqrt{P_{s,m}} h_{sr} x_s + \eta_{r,m}, \quad (1)$$

where $P_{s,m}$ is the source transmitted power at the m -th H-ARQ (re)transmission round, h_{sr} is the coefficient of the source-relay channel, x_s is the transmitted information packet from the source, and $\eta_{r,m}$ is additive noise. If the relay is not involved in forwarding, the received signal $y_{d,m}$ at the destination at the m -th H-ARQ (re)transmission round is

$$y_{d,m} = \sqrt{P_{s,m}} h_{sd} x_s + \eta_{d,m}, \quad (2)$$

where h_{sd} is the source-destination channel coefficient.

If the relay decodes the packet from the source successfully, it helps in forwarding it to the destination using the Alamouti scheme. It is assumed that the relay knows the codeword of the packet. Specifically, if the packet is partitioned into two parts as $x_s = [x_{s,1} \ x_{s,2}]$, then the relay forwards a corresponding vector $x_r = [-x_{s,2}^* \ x_{s,1}^*]$. The received signal $y_{d,m}$ at the destination at the m -th H-ARQ (re)transmission round with relay forwarding can be written as

$$y_{d,m} = \sqrt{P_{s,m}} h_{sd} x_s + \sqrt{P_r} h_{rd} x_r + \eta_{d,m}, \quad (3)$$

where P_r is the relay transmitted power which is assumed to be fixed over all retransmission rounds¹ and h_{rd} is the channel coefficient from the relay to the destination. At the destination, the packet x_s can be recovered based on the orthogonal structure of the Alamouti code. The destination combines the received signals from all (re)transmission rounds and jointly decodes the information packet. We consider quasi-static Rayleigh fading channels, i.e., the channel coefficients are assumed to be fixed during (re)transmissions of a packet and may change independently when a new information packet is transmitted. The channel state information is assumed to be known at the receiver side and unknown at the transmitter side. The channel coefficients h_{sd} , h_{sr} and h_{rd} are modeled as independent, zero-mean complex Gaussian random variables with variances $1/\lambda_{sd}$, $1/\lambda_{sr}$ and $1/\lambda_{rd}$, respectively. The noise variances $\eta_{r,m}$ and $\eta_{d,m}$ are modeled as zero-mean complex Gaussian random variables with variance \mathcal{N}_0 .

3.1.2 Average Total Transmission Power of the Protocol

In this subsection, we first study the overall SNR at the relay and at the destination resulting from the cooperative H-ARQ retransmissions. Then, we calculate the probability of the event that the cooperative H-ARQ relaying protocol stops successfully at the l -th ($1 \leq l \leq L$) round. Finally, we are able to derive the average total transmission power of the cooperative H-ARQ protocol.

The relay is to combine the received signals from all previous (re)transmission rounds from the source based on maximal ratio combining (MRC) technique [22] and jointly decode the information packet. Then, the SNR of the MRC output at the relay at the l -th ($1 \leq l \leq L$) round is

$$\gamma_{r,l} = \frac{\sum_{m=1}^l P_{s,m} |h_{sr}|^2}{\mathcal{N}_0}. \quad (4)$$

The destination combines the received signals from all previous rounds from the source and the relay by using the MRC technique and then jointly decodes the information packet. To determine the overall SNR of the combined signal at the destination, we need the following lemma.

Lemma 1 *In the cooperative H-ARQ relaying scheme, if the relay decodes successfully a packet from the source at the k -th ($1 \leq k < L$) (re)transmission round and starts forwarding at round $k + 1$, then the overall SNR at the destination at the l -th ($k \leq l \leq L$) round is given by*

$$\gamma_{d,l,k} = \frac{\sum_{m=1}^l P_{s,m} |h_{sd}|^2 + (l - k) P_r |h_{rd}|^2}{\mathcal{N}_0}. \quad (5)$$

Proof : First, let us focus on a single transmission round. Since the packet from the source can be written as $x_s = [x_{s,1} \ x_{s,2}]$ and the relay forwards $x_r = [-x_{s,2}^* \ x_{s,1}^*]$, so the corresponding received signals $[y_{d,m,1} \ y_{d,m,2}]^T$ at the destination at the m -th ($1 \leq m \leq L$) round are

$$\begin{bmatrix} y_{d,m,1} \\ y_{d,m,2} \end{bmatrix} = \begin{bmatrix} \sqrt{P_{s,m}} x_{s,1} & -\sqrt{P_{r,m}} x_{s,2}^* \\ \sqrt{P_{s,m}} x_{s,2} & \sqrt{P_{r,m}} x_{s,1}^* \end{bmatrix} \begin{bmatrix} h_{sd} \\ h_{rd} \end{bmatrix} + \begin{bmatrix} \eta_{d,m,1} \\ \eta_{d,m,2} \end{bmatrix},$$

where

$$P_{r,m} = \begin{cases} 0, & \text{if } m \leq k; \\ P_r, & \text{if } m > k. \end{cases}$$

(Recall that the relay is assumed to decode correctly at the k -th round and starts forwarding at the $(k + 1)$ -th round with fixed transmission power.)

¹For simplicity in the treatment herein as well as practical system implementation purposes, it is assumed that relay helpers have fixed, non varying, power level across retransmissions.

The received signal vector can be rewritten as

$$\underbrace{\begin{bmatrix} y_{d,m,1} \\ y_{d,m,2}^* \end{bmatrix}}_{\triangleq Y_m} = \underbrace{\begin{bmatrix} \sqrt{P_{s,m}}h_{sd} & -\sqrt{P_{r,m}}h_{rd} \\ \sqrt{P_{r,m}}h_{rd}^* & \sqrt{P_{s,m}}h_{sd}^* \end{bmatrix}}_{\triangleq H_m} \underbrace{\begin{bmatrix} x_{s,1} \\ x_{s,2}^* \end{bmatrix}}_{\triangleq \mathbf{x}} + \underbrace{\begin{bmatrix} \eta_{d,m,1} \\ \eta_{d,m,2}^* \end{bmatrix}}_{\triangleq N_m}.$$

Thus, the SNR at the destination at the m -th round is

$$\text{SNR}_m = \frac{\|H_m \mathbf{x}\|_F^2}{\|N_m\|_F^2}, \quad (6)$$

where $\|H_m \mathbf{x}\|_F^2$ is the Frobenius norm (or total power) of $H_m \mathbf{x}$. Note that

$$\|H_m \mathbf{x}\|_F^2 = \text{tr}(\mathbf{x} \mathbf{x}^H H_m^H H_m) = (|x_{s,1}|^2 + |x_{s,2}|^2) (P_{s,m}|h_{sd}|^2 + P_{r,m}|h_{rd}|^2).$$

Let us assume that the length and (normalized) power of the packet x_s is \mathcal{J} . Then, we have $\|H_m \mathbf{x}\|_F^2 = \mathcal{J} (P_{s,m}|h_{sd}|^2 + P_{r,m}|h_{rd}|^2)$ and $\|N_m\|_F^2 = \mathcal{J} \mathcal{N}_0$. Thus, the SNR at the destination at the m -th round is

$$\text{SNR}_m = \frac{P_{s,m}|h_{sd}|^2 + P_{r,m}|h_{rd}|^2}{\mathcal{N}_0}, \quad (7)$$

where $P_{r,m} = 0$ if $m \leq k$ and $P_{r,m} = P_r$ if $m > k$. Therefore, the overall SNR at the destination by combining all received signals from the first l rounds is

$$\gamma_{d,l,k} = \sum_{m=1}^l \text{SNR}_m = \sum_{m=1}^k \frac{P_{s,m}|h_{sd}|^2}{\mathcal{N}_0} + \sum_{m=k+1}^l \frac{P_{s,m}|h_{sd}|^2 + P_r|h_{rd}|^2}{\mathcal{N}_0}, \quad (8)$$

which leads to the result in the Lemma. \square

3.1.3 Probability of the Protocol Successfully Completing at the l -th ($1 \leq l \leq L$) Round

When the destination successfully decodes an information packet, the cooperative H-ARQ protocol stops and that may happen at any round. In this subsection, we derive the probability of the event that the protocol stops at the l -th ($1 \leq l \leq L$) round. This probability is a key result in determining the average total transmission power.

Let $\{T_r = k\}$ denote the event that the relay decodes successfully at the k -th round and starts forwarding at round $(k+1)$, for any $k = 1, 2, \dots, l-1$. Especially, let $\{T_r = l\}$ denote the event that the relay decodes unsuccessfully in the first $l-1$ rounds (in this case, the relay has no participation in message forwarding when the protocol finishes at the l -th round). Furthermore, let $p^{\text{stop},1}$ denote the probability that the H-ARQ protocol stops at the first transmission round ($l=1$), which means that the destination decodes successfully at round one. When $2 \leq l \leq L-1$, let $q_{l,k}$ denote the conditional probability that the H-ARQ protocol stops at the l -th round given that the event $\{T_r = k\}$ occurred. In other words, $q_{l,k}$ is the probability of H-ARQ stopping at the l -th round under the condition that the relay started forwarding at the $(k+1)$ -th round for any $k = 1, 2, \dots, l-1$. Especially, let $q_{l,l}$ ($2 \leq l \leq L-1$) denote the probability that the protocol stops successfully at the l -th round before the relay can help. Moreover, when $l = L$, let $q_{L,k}$ denote the conditional probability that the protocol stops at the L -th round no matter whether decoding at the L -th round is successful or not under the condition that the relay started forwarding at the $(k+1)$ -th round for any $k = 1, 2, \dots, L-1$. Especially, let $q_{L,L}$ denote the probability that the protocol stops at the last round regardless of successful decoding or not (in this case again the relay has no chance to help).

With the above notation, the average total transmission power of the cooperative H-ARQ relaying protocol can be given by

$$\bar{P}(L) = P_{s,1} p^{\text{stop},1} + \sum_{l=2}^L \sum_{k=1}^l q_{l,k} \Pr[T_r = k] R_{l,k}, \quad (9)$$

where $R_{l,k}$ is the transmission power that the source and the relay spend totally up to the l -th round, which is given by

$$R_{l,k} = \Omega_l + (l - k)P_r, \quad (10)$$

in which

$$\Omega_l = \sum_{m=1}^l P_{s,m}. \quad (11)$$

First, we calculate the probability $\Pr[T_r = k]$ in (9). For any given targeted SNR γ_0 , the probability that the relay decodes the packet successfully at the first round ($T_r = 1$) is

$$\Pr[T_r = 1] = \Pr\left[\frac{P_{s,1}|h_{sr}|^2}{\mathcal{N}_0} \geq \gamma_0\right] = e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{P_{s,1}}}, \quad (12)$$

where $\tilde{\gamma}_0 \triangleq \gamma_0 \mathcal{N}_0$. For any $T_r = k$, $k = 2, 3, \dots, l-1$, the probability that the relay decodes successfully at the k -th round, which means that the overall SNR is below the targeted SNR γ_0 until the $(k-1)$ -th round but above γ_0 at the subsequent k -th round, is calculated as

$$\begin{aligned} \Pr[T_r = k] &= \Pr[\gamma_{r,k-1} < \gamma_0, \gamma_{r,k} \geq \gamma_0] \\ &= \Pr\left[\gamma_0 - \frac{P_{s,k}|h_{sr}|^2}{\mathcal{N}_0} \leq \gamma_{r,k-1} < \gamma_0\right] \\ &= \Pr[\gamma_{r,k-1} < \gamma_0] - \Pr[\gamma_{r,k} < \gamma_0] \\ &= e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}}. \end{aligned} \quad (13)$$

When $T_r = l$, we have

$$\Pr[T_r = l] = \Pr[\gamma_{r,l-1} < \gamma_0] = 1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-1}}}. \quad (14)$$

Next, we calculate the conditional probability $q_{l,k}$ in (9). When $l = 1$, the probability that the protocol stops at the first round is

$$p^{stop,1} = \Pr\left[\frac{P_{s,1}|h_{sd}|^2}{\mathcal{N}_0} \geq \gamma_0\right] = e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}}. \quad (15)$$

When $2 \leq l \leq L-1$, we consider two scenarios: (i) $k = 1, 2, \dots, l-1$, and (ii) $k = l$. For any $k = 1, 2, \dots, l-1$, the conditional probability $q_{l,k}$ is given by

$$q_{l,k} = \Pr[\gamma_{d,l-1,k} < \gamma_0, \gamma_{d,l,k} \geq \gamma_0] = \Pr[a \leq |h_{sd}|^2 < b], \quad (16)$$

where

$$a = \frac{\tilde{\gamma}_0 - (l-k)P_r|h_{rd}|^2}{\Omega_l}, \quad b = \frac{\tilde{\gamma}_0 - (l-k-1)P_r|h_{rd}|^2}{\Omega_{l-1}}.$$

We observe that b should not be negative, i.e.,

$$b = \frac{\tilde{\gamma}_0 - (l-k-1)P_r|h_{rd}|^2}{\Omega_{l-1}} \geq 0,$$

which implies that $|h_{rd}|^2 \leq \frac{\tilde{\gamma}_0}{(l-k-1)P_r}$. When $a < 0$, it means $|h_{rd}|^2 > \frac{\tilde{\gamma}_0}{(l-k)P_r}$. When $a \geq 0$, it means $|h_{rd}|^2 \leq \frac{\tilde{\gamma}_0}{(l-k)P_r}$. Therefore, the conditional probability (16) can be calculated as follows

$$\begin{aligned} q_{l,k} &= \left[\int_{|h_{rd}|^2=0}^{\frac{\tilde{\gamma}_0}{(l-k)P_r}} \int_{|h_{sd}|^2=a}^b + \int_{|h_{rd}|^2=\frac{\tilde{\gamma}_0}{(l-k)P_r}}^{\frac{\tilde{\gamma}_0}{(l-k-1)P_r}} \int_{|h_{sd}|^2=0}^b \right] \lambda_{sd} e^{-\lambda_{sd}|h_{sd}|^2} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} d|h_{sd}|^2 d|h_{rd}|^2 \\ &\triangleq A_1 + A_2 - A_3, \end{aligned} \quad (17)$$

where

$$\begin{aligned}
A_1 &= \int_0^{\frac{\tilde{\gamma}_0}{(l-k)P_r}} e^{-\lambda_{sd}a} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathbf{d}|h_{rd}|^2 \\
&= \frac{\lambda_{rd}}{\frac{\lambda_{sd}(l-k)P_r}{\Omega_l} - \lambda_{rd}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} \right), \tag{18}
\end{aligned}$$

$$\begin{aligned}
A_2 &= \int_{\frac{\tilde{\gamma}_0}{(l-k)P_r}}^{\frac{\tilde{\gamma}_0}{(l-k-1)P_r}} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathbf{d}|h_{rd}|^2 \\
&= e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k)P_r}} - e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k-1)P_r}}, \tag{19}
\end{aligned}$$

$$\begin{aligned}
A_3 &= \int_0^{\frac{\tilde{\gamma}_0}{(l-k-1)P_r}} e^{-\lambda_{sd}b} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathbf{d}|h_{rd}|^2 \\
&= \frac{\lambda_{rd}}{\frac{\lambda_{sd}(l-k-1)P_r}{\Omega_{l-1}} - \lambda_{rd}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(l-k-1)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right). \tag{20}
\end{aligned}$$

In case of $k = l$, the conditional probability $q_{l,l}$ is given by

$$\begin{aligned}
q_{l,l} &= \Pr[\gamma_{d,l-1,l-1} < \gamma_0, \gamma_{d,l,l} \geq \gamma_0] \\
&= \Pr\left[\frac{\tilde{\gamma}_0}{\Omega_l} \leq |h_{sd}|^2 < \frac{\tilde{\gamma}_0}{\Omega_{l-1}}\right] \\
&= e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}}. \tag{21}
\end{aligned}$$

When $l = L$, to calculate the conditional probability $q_{L,k}$, we also consider two scenarios: (i) $k = 1, 2, \dots, L-1$, and (ii) $k = L$. For any $k = 1, 2, \dots, L-1$, the conditional probability $q_{L,k}$ is given by

$$q_{L,k} = \Pr[\gamma_{d,L-1,k} < \gamma_0] = \Pr[|h_{sd}|^2 < c], \tag{22}$$

where

$$c = \frac{\tilde{\gamma}_0 - (L-k-1)P_r|h_{rd}|^2}{\Omega_{L-1}}.$$

Note that c should not be negative, i.e., $c \geq 0$, which implies that $|h_{rd}|^2 \leq \frac{\tilde{\gamma}_0}{(L-k-1)P_r}$. Therefore, the conditional probability (22) can be calculated as follows

$$q_{L,k} = \int_{|h_{rd}|^2=0}^{\frac{\tilde{\gamma}_0}{(L-k-1)P_r}} \int_{|h_{sd}|^2=0}^c \lambda_{sd} e^{-\lambda_{sd}|h_{sd}|^2} \lambda_{rd} e^{-\lambda_{rd}|h_{rd}|^2} \mathbf{d}|h_{sd}|^2 \mathbf{d}|h_{rd}|^2 \triangleq B_{L-1,k}, \tag{23}$$

where

$$B_{i,j} = 1 - e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(i-j)P_r}} - \frac{\lambda_{rd}}{\frac{\lambda_{sd}(i-j)P_r}{\Omega_i} - \lambda_{rd}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{(i-j)P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_i}} \right). \tag{24}$$

For $k = L$, the conditional probability $q_{L,L}$ is given by

$$q_{L,L} = \Pr[\gamma_{d,L-1,L-1} < \gamma_0] = 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}. \tag{25}$$

Based on the above analysis, the average total transmission power in (9) for the cooperative H-ARQ relaying protocol can be given specifically in the following theorem.

Theorem 1 *In the cooperative H-ARQ relaying protocol, the average total transmission power is*

$$\bar{P}(L) = P_{s,1} e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} + \sum_{l=2}^{L-1} \left(\sum_{k=1}^{l-1} C_{l,k} + C_{l,l} \right) + \left(\sum_{k=1}^{L-1} D_k + D_L \right), \tag{26}$$

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where

$$\begin{aligned}
C_{l,k} &= q_{l,k} \Pr[T_r = k] R_{l,k} \\
&= (A_1 + A_2 - A_3) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) \left[\sum_{m=1}^l P_{s,m} + (l-k)P_r \right], \quad 1 \leq k \leq l-1, \\
C_{l,l} &= q_{l,l} \Pr[T_r = l] R_{l,l} \\
&= \left(e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \sum_{m=1}^l P_{s,m}, \\
D_k &= q_{L,k} \Pr[T_r = k] R_{L,k} \\
&= B_{L-1,k} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) \left[\sum_{m=1}^L P_{s,m} + (L-k)P_r \right], \quad 1 \leq k \leq L-1, \\
D_L &= q_{L,L} \Pr[T_r = L] R_{L,L} \\
&= \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \sum_{m=1}^L P_{s,m},
\end{aligned}$$

in which A_1, A_2 and A_3 are specified in (18)–(20), and $B_{L-1,k}$ is specified in (24), respectively. Also, $\tilde{\gamma}_0 \triangleq \gamma_0 \mathcal{N}_0$, γ_0 is the target SNR, \mathcal{N}_0 is the additive white noise variance, and $1/\lambda_{sd}$, $1/\lambda_{sr}$ and $1/\lambda_{rd}$ are the variances of the source-destination, the source-relay and the relay-destination fading channels, respectively. \square

When $L = 2$, the average total transmission power expression \bar{P} in (26) reduces to

$$\bar{P}(L = 2) = P_{s,1} + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} \right) \left(P_{s,2} + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{P_{s,1}}} P_r \right). \quad (27)$$

3.1.4 Asymptotically Tight Approximation of the Average Total Transmission Power

The general closed-form expression in (26) for the average total transmission power is certainly complicated. In this section, we try to develop a simple and tight approximation of the average total transmission power.

When $L = 2$, if we approximate e^{-x} by $1 - x$ for small x (corresponding to high SNR), then the average total transmission power \bar{P} in (27) can be approximated as

$$\bar{P}(L = 2) \approx P_{s,1} + (P_{s,2} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}, \quad (28)$$

which is tight at high SNR. When $L \geq 3$, however, it is difficult to derive the approximation of $\bar{P}(L \geq 3)$ directly from (26). So, we first reformulate the average total transmission power \bar{P} in (26) which enables us to develop a tight approximation at high SNR.

For simplicity, let us denote $\tilde{C}_{l,k} = q_{l,k} \Pr[T_r = k]$ for $k = 1, 2, \dots, l-1$, and $\tilde{C}_{l,l} = q_{l,l} \Pr[T_r = l]$. Denote $\tilde{D}_k = q_{L,k} \Pr[T_r = k]$ for $k = 1, 2, \dots, L-1$, and $\tilde{D}_L = q_{L,L} \Pr[T_r = L]$. The new notation corresponds to the notation $C_{l,k}$, $C_{l,l}$, D_k , and D_L defined in Theorem 1. We observe that $C_{l,k} = \tilde{C}_{l,k} \left[\sum_{m=1}^l P_{s,m} + (l-k)P_r \right]$, $C_{l,l} = \tilde{C}_{l,l} \sum_{m=1}^l P_{s,m}$, $D_k = \tilde{D}_k \left[\sum_{m=1}^L P_{s,m} + (L-k)P_r \right]$, and $D_L = \tilde{D}_L \sum_{m=1}^L P_{s,m}$. Therefore, the average total transmission power in (26) can be rewritten by switching the summation order (in terms of

$P_{s,m}$, $1 \leq m \leq L$, and P_r first) as follows

$$\begin{aligned}
\bar{P}(L) &= P_{s,1} \left[e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} + \sum_{l=2}^{L-1} \sum_{k=1}^l \tilde{C}_{l,k} + \sum_{k=1}^L \tilde{D}_k \right] \\
&\quad + \sum_{m=2}^{L-1} P_{s,m} \left[\sum_{l=m}^{L-1} \sum_{k=1}^l \tilde{C}_{l,k} + \sum_{k=1}^L \tilde{D}_k \right] + P_{s,L} \sum_{k=1}^L \tilde{D}_k \\
&\quad + P_r \left[\sum_{l=2}^{L-1} \sum_{k=1}^{l-1} (l-k) \tilde{C}_{l,k} + \sum_{k=1}^{L-1} (L-k) \tilde{D}_k \right] \\
&\triangleq \sum_{m=1}^L P_{s,m} Q_m + P_r Q_r,
\end{aligned} \tag{29}$$

where

$$Q_1 = e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}} + \sum_{l=2}^{L-1} \sum_{k=1}^l \tilde{C}_{l,k} + \sum_{k=1}^L \tilde{D}_k, \tag{30}$$

$$Q_m = \sum_{l=m}^{L-1} \sum_{k=1}^l \tilde{C}_{l,k} + \sum_{k=1}^L \tilde{D}_k, \quad 2 \leq m \leq L-1, \tag{31}$$

$$Q_L = \sum_{k=1}^L \tilde{D}_k, \tag{32}$$

$$Q_r = \sum_{l=2}^{L-1} \sum_{k=1}^{l-1} (l-k) \tilde{C}_{l,k} + \sum_{k=1}^{L-1} (L-k) \tilde{D}_k. \tag{33}$$

We observe that

$$Q_1 = Q_2 + e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}}}, \tag{34}$$

and for any $m = 2, 3, \dots, L-1$,

$$Q_m = Q_{m+1} + \sum_{k=1}^m \tilde{C}_{m,k}. \tag{35}$$

In the following, we calculate Q_m for any $m = 1, 2, \dots, L$. Since Q_m can be determined by using the previously determined Q_{m+1} according to (34) and (35), we derive Q_L, Q_{L-1}, \dots, Q_1 in a reverse sequential order as follows. First, Q_L in (32) can be calculated specifically as follows

$$\begin{aligned}
Q_L &= \sum_{k=1}^{L-1} B_{L-1,k} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \\
&= \sum_{k=1}^{L-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-1,k} - B_{L-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right),
\end{aligned} \tag{36}$$

where $B_{i,j}$ is specified in (24). For any $l = L - 1, L - 2, \dots, 3$, Q_l in (35) can be calculated as follows²

$$\begin{aligned}
Q_l &= Q_{l+1} + \sum_{k=1}^{l-1} (A_1 + A_2 - A_3) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) + \left(e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \\
&= Q_{l+1} - \left[\sum_{k=1}^{l-1} B_{l,k} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \right] \\
&\quad + \left[\sum_{k=1}^{l-1} B_{l-1,k} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \right] \\
&= Q_{l+1} - \left[\sum_{k=1}^{l-1} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l,k} - B_{l,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_l}} \right) \right] \\
&\quad + \left[\sum_{k=1}^{l-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l-1,k} - B_{l-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right) \right], \tag{37}
\end{aligned}$$

in which A_1, A_2 and A_3 are specified in (18)–(20), and $B_{i,j}$ is specified in (24), respectively. When $l = L - 1$, substituting Q_L in (36) into (37), we have

$$\begin{aligned}
Q_{L-1} &= Q_L - \underbrace{\left[\sum_{k=1}^{L-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-1,k} - B_{L-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \right]}_{=0} \\
&\quad + \left[\sum_{k=1}^{L-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-2,k} - B_{L-2,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-2}}} \right) \right] \\
&= \sum_{k=1}^{L-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-2,k} - B_{L-2,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-2}}} \right). \tag{38}
\end{aligned}$$

When $l = L - 2$, substituting the above derivation (38) into (37), we can calculate Q_{L-2} as

$$\begin{aligned}
Q_{L-2} &= Q_{L-1} - \underbrace{\left[\sum_{k=1}^{L-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-2,k} - B_{L-2,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-2}}} \right) \right]}_{=0} \\
&\quad + \left[\sum_{k=1}^{L-4} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-3,k} - B_{L-3,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-3}}} \right) \right] \\
&= \sum_{k=1}^{L-4} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-3,k} - B_{L-3,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-3}}} \right). \tag{39}
\end{aligned}$$

Assume that for any $l \geq l_0 (< L)$, it is true that

$$Q_l = \sum_{k=1}^{l-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l-1,k} - B_{l-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right), \quad l = L - 1, L - 2, \dots, l_0, \tag{40}$$

²To maintain the simplicity of the notation, we change subscript m in (35) into l .

then for $l = l_0 - 1$, we have

$$\begin{aligned}
Q_{l_0-1} &= Q_{l_0} - \underbrace{\left[\sum_{k=1}^{l_0-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l_0-1,k} - B_{l_0-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l_0-1}}}\right) \right]}_{=0} \\
&\quad + \left[\sum_{k=1}^{l_0-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l_0-2,k} - B_{l_0-2,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l_0-2}}}\right) \right] \\
&= \sum_{k=1}^{l_0-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l_0-2,k} - B_{l_0-2,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l_0-2}}}\right), \tag{41}
\end{aligned}$$

i.e., the result in (40) is also true for $l = l_0 - 1$. Thus, by induction we can conclude that for any $l = L - 1, L - 2, \dots, 3$, we have

$$Q_l = \sum_{k=1}^{l-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l-1,k} - B_{l-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}}\right). \tag{42}$$

When $l = 2$, from $Q_3 = e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}} (B_{2,1} - B_{2,2}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_2}}\right)$, we can calculate Q_2 in the following

$$\begin{aligned}
Q_2 &= Q_3 + \tilde{C}_{2,1} + \tilde{C}_{2,2} \\
&= Q_3 + \frac{\lambda_{sd}}{\lambda_{sd} - \frac{\lambda_{rd}\Omega_2}{P_r}} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}} \left(e^{-\frac{\lambda_{rd}\tilde{\gamma}_0}{P_r}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_2}} \right) + \left(e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_2}} - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}} \right) \\
&= 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}}. \tag{43}
\end{aligned}$$

When $l = 1$, substituting $Q_2 = 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}}$ into (34), we have

$$Q_1 = Q_2 + e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}} = 1. \tag{44}$$

We may further approximate the probability Q_l for any $l = 2, 3, \dots, L$. When $l = 2$, if we approximate e^{-x} by $1 - x$ for small x (high SNR), then Q_2 in (43) can be approximated as

$$Q_2 \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}. \tag{45}$$

Note that if we approximate e^{-x} by $1 - x + \frac{1}{2}x^2$ for small x (high SNR), then for $1 \leq k \leq l-3$, $e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l-1,k} - B_{l-1,k+1})$ in (42) can be approximated as³

$$e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l-1,k} - B_{l-1,k+1}) \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_{l-1}} \frac{\lambda_{rd}\tilde{\gamma}_0}{P_r} \left(\frac{1}{l-1-k} - \frac{1}{l-2-k} \right), \tag{46}$$

and when $k = l-2$,

$$e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}}} (B_{l-1,l-2} - B_{l-1,l-1}) \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \left(-1 + \frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_{l-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2P_r} \right). \tag{47}$$

³When we approximate e^{-x} by the first three terms of its Taylor expansion $1 - x + \frac{1}{2}x^2$ for small x (corresponding to high SNR), then $e^{-x_1} - e^{-x_2}$ can be tightly approximated as $(x_1 - x_2) [-1 + \frac{1}{2}(x_1 + x_2)]$.

Therefore, for any $l = 3, 4, \dots, L$, Q_l in (36) and (42) can be approximated as

$$\begin{aligned}
Q_l &\approx \underbrace{\sum_{k=1}^{l-3} \frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_{l-1}} \frac{\lambda_{rd}\tilde{\gamma}_0}{P_r} \left(\frac{1}{l-1-k} - \frac{1}{l-2-k} \right)}_{= \frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_{l-1}} \frac{\lambda_{rd}\tilde{\gamma}_0}{P_r} \left(\frac{1}{l-2} - 1 \right)} \\
&\quad + \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2P_r} \right) + \underbrace{\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \left(-1 + \frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_{l-1}} \right) + \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} - \frac{1}{2} \left(\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \right)^2}_{= 0} \\
&= \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r} \right). \tag{48}
\end{aligned}$$

The above calculation of the probability Q_l can be summarized in the following lemma.

Lemma 2 *The probability Q_l in (30)–(32) has a closed-form expression as follows:*

$$Q_l = \begin{cases} 1, & l = 1; \\ 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}}, & l = 2; \\ \sum_{k=1}^{l-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{l-1,k} - B_{l-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}}} \right), & 3 \leq l \leq L, \end{cases} \tag{49}$$

where $B_{i,j}$ is specified in (24). Furthermore, the probability Q_l can be tightly approximated by

$$Q_l \approx \begin{cases} \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}, & l = 2; \\ \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r} \right), & 3 \leq l \leq L. \end{cases} \tag{50}$$

Next, we calculate the probability Q_r in (33). To calculate Q_r more efficiently, we partition Q_r as follows (corresponding to Q_2, Q_3, \dots, Q_L)

$$Q_r \triangleq \sum_{i=2}^L Q_{r,i}, \tag{51}$$

where

$$Q_{r,2} = \sum_{l=2}^{L-1} \tilde{C}_{l,1} + \tilde{D}_1, \tag{52}$$

$$Q_{r,i} = \sum_{l=i}^{L-1} \sum_{k=1}^{l-1} \tilde{C}_{l,k} + \sum_{k=1}^{L-2} \tilde{D}_k - \sum_{k=i}^{L-2} \left(\sum_{l=k+1}^{L-1} \tilde{C}_{l,k} + \tilde{D}_k \right), \quad 3 \leq i \leq L-2, \tag{53}$$

$$Q_{r,L-1} = \sum_{k=1}^{L-2} \tilde{C}_{L-1,k} + \sum_{k=1}^{L-2} \tilde{D}_k, \tag{54}$$

$$Q_{r,L} = \sum_{k=1}^{L-1} \tilde{D}_k. \tag{55}$$

We observe that

$$Q_{r,L-1} = Q_{r,L} + \sum_{k=1}^{L-2} \tilde{C}_{L-1,k} - \tilde{D}_{L-1}, \tag{56}$$

and for any $i = 3, 4, \dots, L-2$,

$$Q_{r,i} = Q_{r,i+1} + \sum_{k=1}^{i-1} \tilde{C}_{i,k} - \left(\sum_{l=i+1}^{L-1} \tilde{C}_{l,i} + \tilde{D}_i \right). \quad (57)$$

In the following, we calculate $Q_{r,i}$ for any $i = 2, 3, \dots, L$. Since $Q_{r,i}$ can be determined by using the previously determined $Q_{r,i+1}$ according to (56) and (57), we derive $Q_{r,L}, Q_{r,L-1}, \dots, Q_{r,2}$ in a reverse sequential order as follows. First, $Q_{r,L}$ in (55) can be calculated specifically as follows

$$\begin{aligned} Q_{r,L} &= \sum_{k=1}^{L-1} B_{L-1,k} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) \\ &= \sum_{k=1}^{L-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-1,k} - B_{L-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right), \end{aligned} \quad (58)$$

where $B_{i,j}$ is specified in (24). When $i = L-1$, $Q_{r,L-1}$ in (56) can be elaborated as follows

$$\begin{aligned} Q_{r,L-1} &= Q_{r,L} + \sum_{k=1}^{L-2} (-B_{L-1,k} + B_{L-2,k}) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) - \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-2}}} \right) \\ &= Q_{r,L} - \left[\sum_{k=1}^{L-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-1,k} - B_{L-1,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \right] \\ &\quad + \left[\sum_{k=1}^{L-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-2,k} - B_{L-2,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-2}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-2}}} \right) \right]. \end{aligned} \quad (59)$$

By substituting $Q_{r,L}$ in (58) into (59), we have

$$\begin{aligned} Q_{r,L-1} &= \sum_{k=1}^{L-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-2,k} - B_{L-2,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-2}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-2}}} \right) \\ &\quad - \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right). \end{aligned} \quad (60)$$

For any $i = L-2, L-3, \dots, 3$, $Q_{r,i}$ in (57) can be calculated as follows

$$\begin{aligned} Q_{r,i} &= Q_{r,i+1} + \sum_{k=1}^{i-1} (-B_{i,k} + B_{i-1,k}) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) \\ &\quad - \left(\sum_{l=i+1}^{L-1} (-B_{l,i} + B_{l-1,i}) + B_{L-1,i} \right) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_i}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}} \right) \\ &= Q_{r,i+1} + \sum_{k=1}^{i-1} (-B_{i,k} + B_{i-1,k}) \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) - B_{i,i} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_i}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}} \right) \\ &= Q_{r,i+1} - \left[\sum_{k=1}^{i-1} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i,k} - B_{i,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_i}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_i}} \right) \right] \\ &\quad + \left[\sum_{k=1}^{i-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i-1,k} - B_{i-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}}} \right) \right]. \end{aligned} \quad (61)$$

When $i = L - 2$, substituting $Q_{r,L-1}$ in (60) into (61), we can calculate $Q_{r,L-2}$ as

$$Q_{r,L-2} = \sum_{k=1}^{L-4} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-3,k} - B_{L-3,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-3}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-3}}}\right) - \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\right) \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right). \quad (62)$$

Assume that for any $i \geq i_0 (< L - 1)$, it is true that

$$Q_{r,i} = \sum_{k=1}^{i-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i-1,k} - B_{i-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}}}\right) - \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\right) \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right), \quad i = L - 2, L - 3, \dots, i_0, \quad (63)$$

then for $i = i_0 - 1$, we have

$$\begin{aligned} Q_{r,i_0-1} &= Q_{r,i_0} - \left[\sum_{k=1}^{i_0-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i_0-1,k} - B_{i_0-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i_0-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i_0-1}}}\right) \right] \\ &\quad + \left[\sum_{k=1}^{i_0-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i_0-2,k} - B_{i_0-2,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i_0-2}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i_0-2}}}\right) \right] \\ &= \sum_{k=1}^{i_0-3} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i_0-2,k} - B_{i_0-2,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i_0-2}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i_0-2}}}\right) \\ &\quad - \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\right) \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right), \end{aligned} \quad (64)$$

i.e., the result in (63) is also true for $i = i_0 - 1$. Thus, by induction we can conclude that for any $i = L - 2, L - 3, \dots, 3$, we have

$$Q_{r,i} = \sum_{k=1}^{i-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i-1,k} - B_{i-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}}}\right) - \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}}\right) \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}}\right). \quad (65)$$

When $i = 2$, $Q_{r,2}$ in (52) can be calculated directly as

$$\begin{aligned} Q_{r,2} &= \sum_{l=2}^{L-1} (-B_{l,1} + B_{l-1,1}) e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}} + B_{L-1,1} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}} \\ &= B_{1,1} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}} \\ &= \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}}\right) e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}}. \end{aligned} \quad (66)$$

Furthermore, we can tightly approximate the probability $Q_{r,i}$ for any $i = 2, 3, \dots, L$. When $i = 2$, $Q_{r,2}$ in (66) can be approximated as

$$Q_{r,2} \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} \left(1 - \frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1} - \frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_1}\right) \quad (67)$$

$$\approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}, \quad (68)$$

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in which the terms $\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}$ and $\frac{\lambda_{sd}\tilde{\gamma}_0}{2\Omega_1}$ in (67) are much smaller than 1 at high SNR, which can be ignored. For any $i = 3, 4, \dots, L-1$, in a similar way as Q_l in (48), $Q_{r,i}$ in (60) and (65) can be approximated as

$$Q_{r,i} \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-2}} - \frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-2)P_r} \right) - \frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}} \quad (69)$$

$$\approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-2)P_r} \right), \quad (70)$$

in which the terms $\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}$ and $\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}$ in (69) are much smaller than $\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-2}}$ and $\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}} \frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-2}}$, respectively, at high SNR, which can be further ignored. When $i = L$, $Q_{r,L}$ in (58) can be approximated similarly as

$$Q_{r,L} \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-2)P_r} \right). \quad (71)$$

The above calculation of the probability Q_r can be summarized in the following lemma.

Lemma 3 *The probability Q_r in (33) has a closed-form expression as follows:*

$$Q_r = \sum_{i=2}^L Q_{r,i}, \quad (72)$$

where

$$Q_{r,i} = \begin{cases} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_1}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1}} \right), & i = 2; \\ \sum_{k=1}^{i-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{i-1,k} - B_{i-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}}} \right) \\ \quad - \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right), & 3 \leq i \leq L-1; \\ \sum_{k=1}^{L-2} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L-1,k} - B_{L-1,k+1}) + e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}}} \right), & i = L, \end{cases} \quad (73)$$

in which $B_{i,j}$ is specified in (24). Furthermore, the probability Q_r can be tightly approximated by

$$Q_r \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{i=3}^L \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{i-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-2)P_r} \right). \quad (74)$$

Based on Lemmas 2 and 3, the average total transmission power $\bar{P}(L)$ in (29) can be approximated as follows.

Theorem 2 *In the cooperative H-ARQ relaying protocol, the average total transmission power can be tightly approximated at high SNR scenario as*

$$\bar{P}(L) \approx P_{s,1} + (P_{s,2} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{P_{s,1}} + \sum_{l=3}^L (P_{s,l} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\sum_{m=1}^{l-1} P_{s,m}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\sum_{m=1}^{l-2} P_{s,m}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r} \right). \quad (75)$$

□

3.1.5 Optimal Transmission Power Assignment for the Cooperative ARQ Protocol

In this subsection, we determine an optimal power assignment strategy for the cooperative H-ARQ relaying protocol based on the asymptotically tight approximation of the average total power consumption developed in the previous section. We derive a set of equations that describe the optimal power level in each (re)transmission round and enable its recursive calculation with fixed searching complexity.

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Before we formulate the problem of finding the optimal transmission power sequence, we need to derive the outage probability of the cooperative H-ARQ relaying protocol with maximum L (re)transmission rounds. Let $p_{T_r=k}^{out,L}$ denote the conditional probability that the destination decodes an information packet unsuccessfully after L (re)transmission rounds given that the event $\{T_r = k\}$ occurred. In other words, $p_{T_r=k}^{out,L}$ is the outage probability at the destination in spite of the fact that the relay started forwarding at the $(k+1)$ -th round for any $k = 1, 2, \dots, L-1$. Therefore, the outage probability of the cooperative H-ARQ relaying scheme after L (re)transmission rounds can be represented as

$$p^{out,L} = \sum_{k=1}^L p_{T_r=k}^{out,L} \cdot \Pr[T_r = k], \quad (76)$$

where the probability $\Pr[T_r = k]$ is specified in (12)–(14). This outage probability is a necessary condition to determine the optimal transmission power sequence. The conditional outage probability $p_{T_r=k}^{out,L}$ in (76) can be evaluated as

$$p_{T_r=k}^{out,L} = \Pr[\gamma_{d,L,k} < \gamma_0], \quad (77)$$

where the overall SNR $\gamma_{d,L,k}$ is specified in (5). When $T_r = L$, we have

$$p_{T_r=L}^{out,L} = \Pr[\gamma_{d,L,L} < \gamma_0] = 1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L}}. \quad (78)$$

For any $T_r = k$, $k = 1, 2, \dots, L-1$, the conditional outage probability is given by

$$p_{T_r=k}^{out,L} = \Pr\left[|h_{sd}|^2 < \frac{\tilde{\gamma}_0 - (L-k)P_r|h_{rd}|^2}{\Omega_L}\right] = B_{L,k}, \quad (79)$$

where $B_{i,j}$ is specified in (24). Finally, based on the probability $\Pr[T_r = k]$ in (12)–(14) and the conditional outage probability $p_{T_r=k}^{out,L}$ in (78) and (79), we can obtain the outage probability for the cooperative H-ARQ relaying scheme after L (re)transmission rounds as follows

$$\begin{aligned} p^{out,L} &= \sum_{k=1}^{L-1} B_{L,k} \left(e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{k-1}}} \right) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L}} \right) \left(1 - e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}}} \right) \\ &= \sum_{k=1}^{L-1} e^{-\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_k}} (B_{L,k} - B_{L,k+1}) + \left(1 - e^{-\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L}} \right). \end{aligned} \quad (80)$$

We can further approximate $p^{out,L}$ in (80) for high SNR scenario as

$$p^{out,L} \approx \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r} \right). \quad (81)$$

In the following, we determine an optimal power sequence $\mathbf{P} = [P_{s,1}, P_{s,2}, \dots, P_{s,L}; P_r]$ for the cooperative H-ARQ relaying protocol such that the average total transmission power for the protocol to deliver an information packet is minimized. We assume that the relay transmitted power P_r is fixed over all retransmission rounds. For the H-ARQ relay protocol with a targeted outage probability p_0 , the problem of finding optimal power assignment per round can be formulated as

$$\begin{aligned} \min \quad & \bar{P} \quad \text{with respect to } P_{s,1}, P_{s,2}, \dots, P_{s,L}; P_r \geq 0 \\ \text{subject to} \quad & p^{out,L} \leq p_0 \end{aligned} \quad (82)$$

where \bar{P} and $p^{out,L}$ are specified in (26) and (80), respectively. Since solving the optimization problem with the closed-form expressions in (26) and (80) is not analytically tractable, we use the asymptotically tight approximation results of the average total transmission power and the outage probability, which are derived in (75) and

(81), respectively. Then, the optimization problem is given by

$$\begin{aligned} \min_{P_{s,1}, \dots, P_{s,L}; P_r \geq 0} \quad & P_{s,1} + (P_{s,2} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^L (P_{s,l} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r} \right) \\ \text{subject to} \quad & \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r} \right) \leq p_0 \end{aligned} \quad (83)$$

where $\Omega_l = \sum_{m=1}^l P_{s,m}$.

Before we solve the above optimization problem, we first show that the average total transmission power \bar{P} reaches its minimum when the constraint in (83) holds with equality. If there exists a power sequence $P_{s,1}^*, P_{s,2}^*, \dots, P_{s,L}^*, P_r^*$ such that $\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L^*} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}^*} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r^*} \right) < p_0$ where $\Omega_l^* = \sum_{m=1}^l P_{s,m}^*$, and the average total transmission power \bar{P} is minimized, then let us consider another power sequence

$$\begin{cases} \tilde{P}_{s,m} = P_{s,m}^*, & 1 \leq m \leq L-1, \\ \tilde{P}_{s,L} = \eta P_{s,L}^*, \\ \tilde{P}_r = P_r^*, \end{cases}$$

where η is

$$\eta \triangleq \frac{1}{P_{s,L}^*} \left[\frac{\lambda_{sd}\tilde{\gamma}_0}{p_0} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}^*} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r^*} \right) - \Omega_{L-1}^* \right]. \quad (84)$$

We can see that the new power sequence $\tilde{P}_{s,1}, \tilde{P}_{s,2}, \dots, \tilde{P}_{s,L}, \tilde{P}_r$ satisfies

$$\frac{\lambda_{sd}\tilde{\gamma}_0}{\tilde{\Omega}_L} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\tilde{\Omega}_{L-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)\tilde{P}_r} \right) = p_0, \quad (85)$$

where $\tilde{\Omega}_l = \sum_{m=1}^l \tilde{P}_{s,m}$. Since $\frac{\lambda_{sd}\tilde{\gamma}_0}{p_0} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}^*} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r^*} \right) < \Omega_L^*$, so η is upper bounded as

$$\eta < \frac{1}{P_{s,L}^*} [\Omega_L^* - \Omega_{L-1}^*] = 1, \quad (86)$$

and the corresponding average total transmission power is

$$\begin{aligned} \bar{P}(\tilde{P}_{s,1}, \tilde{P}_{s,2}, \dots, \tilde{P}_{s,L}; \tilde{P}_r) &= P_{s,1}^* + (P_{s,2}^* + P_r^*) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1^*} \\ &\quad + \sum_{l=3}^{L-1} (P_{s,l}^* + P_r^*) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{l-1}^*} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{l-2}^*} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(l-2)P_r^*} \right) \\ &\quad + (\eta P_{s,L}^* + P_r^*) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_{L-1}^*} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-2}^*} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-2)P_r^*} \right) \\ &< \bar{P}(P_{s,1}^*, P_{s,2}^*, \dots, P_{s,L}^*; P_r^*), \end{aligned} \quad (87)$$

which implies that the average total transmission power resulting from the new power sequence $\tilde{P}_{s,1}, \tilde{P}_{s,2}, \dots, \tilde{P}_{s,L}, \tilde{P}_r$ is less than that based on the power sequence $P_{s,1}^*, P_{s,2}^*, \dots, P_{s,L}^*, P_r^*$. This is contradictory to the assumption that the power sequence $P_{s,1}^*, P_{s,2}^*, \dots, P_{s,L}^*, P_r^*$ minimizes the average total transmission power \bar{P} . Therefore, the minimum average total transmission power \bar{P} can be achieved at the boundary of the constraint (with equality) in (83).

Next, based on the constraint in (83) with equality, we solve the optimization problem by considering a Lagrange multiplier method. Denote $F_i \triangleq \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_i} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{i-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(i-1)P_r} \right)$, then a Lagrangian objective function can be formulated as

$$\mathcal{L}(\mathbf{P}, \lambda) = P_{s,1} + (P_{s,2} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^L (P_{s,l} + P_r) F_{l-1} + \lambda [F_L - p_0]. \quad (88)$$

Further denote $G_i \triangleq \frac{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}{\Omega_{i-1}^2}$ and $M_i \triangleq \frac{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}{2(i-1)P_r^2}$, then the derivatives of $\mathcal{L}(\mathbf{P}, \lambda)$ with respect to $P_{s,m}$, $1 \leq m \leq L$, and P_r are

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{L}}{\partial P_{s,1}} = 1 - (P_{s,2} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1^2} - \sum_{l=3}^L (P_{s,l} + P_r) \frac{F_{l-1} + G_{l-1}}{\Omega_{l-1}} - \lambda \frac{F_L + G_L}{\Omega_L}, \\ \frac{\partial \mathcal{L}}{\partial P_{s,2}} = \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (P_{s,3} + P_r) \frac{F_2}{\Omega_2} - \sum_{l=4}^L (P_{s,l} + P_r) \frac{F_{l-1} + G_{l-1}}{\Omega_{l-1}} - \lambda \frac{F_L + G_L}{\Omega_L}, \\ \frac{\partial \mathcal{L}}{\partial P_{s,m}} = F_{m-1} - (P_{s,m+1} + P_r) \frac{F_m}{\Omega_m} - \sum_{l=m+2}^L (P_{s,l} + P_r) \frac{F_{l-1} + G_{l-1}}{\Omega_{l-1}} - \lambda \frac{F_L + G_L}{\Omega_L}, \\ \hspace{25em} m = 3, 4, \dots, L-2, \\ \frac{\partial \mathcal{L}}{\partial P_{s,L-1}} = F_{L-2} - (P_{s,L} + P_r) \frac{F_{L-1}}{\Omega_{L-1}} - \lambda \frac{F_L + G_L}{\Omega_L}, \\ \frac{\partial \mathcal{L}}{\partial P_{s,L}} = F_{L-1} - \lambda \frac{F_L}{\Omega_L}, \\ \frac{\partial \mathcal{L}}{\partial P_r} = \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^L \left[F_{l-1} - (P_{s,l} + P_r) \frac{M_{l-1}}{\Omega_{l-1}} \right] - \lambda \frac{M_L}{\Omega_L}. \end{array} \right.$$

Based on $\frac{\partial \mathcal{L}}{\partial P_{s,1}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_{s,2}} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_{s,1}} - \frac{\partial \mathcal{L}}{\partial P_{s,2}} = \frac{1}{\Omega_1} \left[\Omega_1 - (\Omega_2 + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (P_{s,3} + P_r) \frac{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}{\Omega_1\Omega_2} \right] = 0,$$

which implies

$$P_{s,3} = \frac{\Omega_1\Omega_2}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2} \left[\Omega_1 - (\Omega_2 + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} \right] - P_r. \quad (89)$$

Based on $\frac{\partial \mathcal{L}}{\partial P_{s,2}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_{s,3}} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_{s,2}} - \frac{\partial \mathcal{L}}{\partial P_{s,3}} = \frac{1}{\Omega_2} \left[\Omega_2 \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (\Omega_3 + P_r) F_2 - (P_{s,4} + P_r) \frac{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}{\Omega_2\Omega_3} \right] = 0,$$

which implies

$$P_{s,4} = \frac{\Omega_2\Omega_3}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2} \left[\Omega_2 \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (\Omega_3 + P_r) F_2 \right] - P_r. \quad (90)$$

For any $m = 5, 6, \dots, L$, according to $\frac{\partial \mathcal{L}}{\partial P_{s,m-2}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_{s,m-1}} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_{s,m-2}} - \frac{\partial \mathcal{L}}{\partial P_{s,m-1}} = \frac{1}{\Omega_{m-2}} \left[\Omega_{m-2} F_{m-3} - (\Omega_{m-1} + P_r) F_{m-2} - (P_{s,m} + P_r) \frac{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2}{\Omega_{m-2}\Omega_{m-1}} \right] = 0,$$

which implies

$$P_{s,m} = \frac{\Omega_{m-2}\Omega_{m-1}}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0^2} \left[\Omega_{m-2} F_{m-3} - (\Omega_{m-1} + P_r) F_{m-2} \right] - P_r. \quad (91)$$

Moreover, by taking the derivative of $\mathcal{L}(\mathbf{P}, \lambda)$ with respect to λ and setting it equal to zero, we have the constraint as $F_L - p_0 = 0$. Furthermore, from $\frac{\partial \mathcal{L}}{\partial P_{s,L}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_r} = 0$, we have constraints as $F_{L-1} - \lambda \frac{F_L}{\Omega_L} = 0$ and $\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^L \left[F_{l-1} - (P_{s,l} + P_r) \frac{M_{l-1}}{\Omega_{l-1}} \right] - \lambda \frac{M_L}{\Omega_L} = 0$, respectively. Based on these three constraints, we come up with

$$\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^L \left[F_{l-1} - (P_{s,l} + P_r) \frac{M_{l-1}}{\Omega_{l-1}} \right] = \frac{F_{L-1}M_L}{p_0}. \quad (92)$$

We summarize the above discussion in the following theorem which shows how to determine the optimal transmission power sequence $P_{s,1}, P_{s,2}, \dots, P_{s,L}$ and P_r .

Theorem 3 In the cooperative H-ARQ relaying protocol, to minimize the average total transmission power, the optimal transmission power sequence $P_{s,1}, P_{s,2}, \dots, P_{s,L}$ and P_r satisfies the following

$$P_{s,3} = \frac{\Omega_1 \Omega_2}{\lambda_{sd} \lambda_{sr} \tilde{\gamma}_0^2} \left[\Omega_1 - (\Omega_2 + P_r) \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1} \right] - P_r, \quad (93)$$

$$P_{s,4} = \frac{\Omega_2 \Omega_3}{\lambda_{sd} \lambda_{sr} \tilde{\gamma}_0^2} \left[\Omega_2 \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1} - (\Omega_3 + P_r) F_2 \right] - P_r, \quad (94)$$

$$P_{s,l} = \frac{\Omega_{l-2} \Omega_{l-1}}{\lambda_{sd} \lambda_{sr} \tilde{\gamma}_0^2} \left[\Omega_{l-2} F_{l-3} - (\Omega_{l-1} + P_r) F_{l-2} \right] - P_r, \quad l = 5, 6, \dots, L \quad (95)$$

and

$$\frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1} + \sum_{l=3}^L \left[F_{l-1} - (P_{s,l} + P_r) \frac{M_{l-1}}{\Omega_{l-1}} \right] = \frac{F_{L-1} M_L}{p_0}, \quad (96)$$

where

$$F_i = \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_i} \left(\frac{\lambda_{sr} \tilde{\gamma}_0}{\Omega_{i-1}} + \frac{\lambda_{rd} \tilde{\gamma}_0}{2(i-1)P_r} \right), \quad M_i = \frac{\lambda_{sd} \lambda_{rd} \tilde{\gamma}_0^2}{2(i-1)P_r^2},$$

in which $\Omega_i = \sum_{m=1}^i P_{s,m}$, $\tilde{\gamma}_0 \triangleq \gamma_0 \mathcal{N}_0$, γ_0 is the target SNR, \mathcal{N}_0 is the additive white noise variance, and $1/\lambda_{sd}$, $1/\lambda_{sr}$ and $1/\lambda_{rd}$ are the variances of the source-destination, the source-relay and the relay-destination fading channels, respectively. \square

From Theorem 3, we observe that for any given power P_r , $P_{s,1}$ and $P_{s,2}$, we can determine power $P_{s,3}$. In general, for any $l = 3, 4, \dots, L$, with power P_r and $P_{s,1}, P_{s,2}, \dots, P_{s,l-1}$, we can determine power $P_{s,l}$. Thus, for any given initial power P_r , $P_{s,1}$ and $P_{s,2}$, we can determine power $P_{s,l}$ recursively for any $l = 3, 4, \dots, L$. The complexity of finding the optimal power sequence is reduced to that of searching the initial power $P_{s,1}$, $P_{s,2}$ and P_r , i.e., searching over a three-variable space. We can see that in such a way, the calculation complexity of finding the optimal power sequence for the cooperative H-ARQ relaying protocol is fixed regardless of the maximum number of (re)transmission rounds L .

Especially, when $L = 2$, we can solve the equations directly and obtain closed-form results. In this case, a Lagrangian objective function can be formulated as

$$\mathcal{L}(\mathbf{P}, \lambda) = P_{s,1} + (P_{s,2} + P_r) \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1} + \lambda [F_2 - p_0]. \quad (97)$$

The derivatives of $\mathcal{L}(\mathbf{P}, \lambda)$ with respect to $P_{s,1}$, $P_{s,2}$, P_r and λ are

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial P_{s,1}} = 1 - (P_{s,2} + P_r) \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1^2} - \lambda \frac{F_2 + G_2}{\Omega_2}, \\ \frac{\partial \mathcal{L}}{\partial P_{s,2}} = \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1} - \lambda \frac{F_2}{\Omega_2}, \\ \frac{\partial \mathcal{L}}{\partial P_r} = \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_1} - \lambda \frac{M_2}{\Omega_2}, \\ \frac{\partial \mathcal{L}}{\partial \lambda} = F_2 - p_0, \end{cases}$$

where

$$F_2 = \frac{\lambda_{sd} \tilde{\gamma}_0}{\Omega_2} \left(\frac{\lambda_{sr} \tilde{\gamma}_0}{\Omega_1} + \frac{\lambda_{rd} \tilde{\gamma}_0}{2P_r} \right), \quad G_2 = \frac{\lambda_{sd} \lambda_{sr} \tilde{\gamma}_0^2}{\Omega_1^2}, \quad M_2 = \frac{\lambda_{sd} \lambda_{rd} \tilde{\gamma}_0^2}{2P_r^2}.$$

From $\frac{\partial \mathcal{L}}{\partial P_r} = 0$, we have

$$\lambda = \frac{2P_r^2 \Omega_2}{\lambda_{rd} \tilde{\gamma}_0 \Omega_1}. \quad (98)$$

Based on $\frac{\partial \mathcal{L}}{\partial P_{s,1}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_{s,2}} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_{s,1}} - \frac{\partial \mathcal{L}}{\partial P_{s,2}} = 1 - \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1} - (P_{s,2} + P_r) \frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_1^2} - \lambda \frac{G_2}{\Omega_2} = 0. \quad (99)$$

By substituting (98) into (99), we have

$$\frac{2\lambda_{sr}}{\lambda_{rd}} P_r^2 + P_{s,1}P_r + P_{s,1}P_{s,2} + P_{s,1}^2 - \frac{P_{s,1}^3}{\lambda_{sd}\tilde{\gamma}_0} = 0. \quad (100)$$

Based on $\frac{\partial \mathcal{L}}{\partial P_{s,2}} = 0$ and $\frac{\partial \mathcal{L}}{\partial P_r} = 0$, we have

$$\frac{\partial \mathcal{L}}{\partial P_{s,2}} - \frac{\partial \mathcal{L}}{\partial P_r} = -\lambda \frac{F_2 - M_2}{\Omega_2} = 0,$$

which implies

$$\frac{2\lambda_{sr}}{\lambda_{rd}} P_r^2 + P_{s,1}P_r - P_{s,1}(P_{s,1} + P_{s,2}) = 0. \quad (101)$$

From (100) and (101), we have

$$P_{s,2} = P_{s,1} \left(\frac{P_{s,1}}{2\lambda_{sd}\tilde{\gamma}_0} - 1 \right) \approx \frac{P_{s,1}^2}{2\lambda_{sd}\tilde{\gamma}_0}, \quad (102)$$

where the term $\frac{P_{s,1}}{2\lambda_{sd}\tilde{\gamma}_0}$ in (102) is much larger than 1 at high SNR, in which 1 can be ignored. By substituting (102) into (101), we have

$$P_r = \frac{\lambda_{rd}P_{s,1}}{4\lambda_{sr}} \left(\sqrt{1 + \frac{4\lambda_{sr}P_{s,1}}{\lambda_{sd}\lambda_{rd}\tilde{\gamma}_0}} - 1 \right) \approx \frac{P_{s,1}}{2} \sqrt{\frac{\lambda_{rd}P_{s,1}}{\lambda_{sd}\lambda_{sr}\tilde{\gamma}_0}}, \quad (103)$$

where the term $\frac{4\lambda_{sr}P_{s,1}}{\lambda_{sd}\lambda_{rd}\tilde{\gamma}_0}$ in (103) is much larger than 1 at high SNR, in which all 1's can be ignored. By substituting (102) and (103) into $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$, we have

$$P_{s,1}^3 = \frac{2\lambda_{sd}^2\lambda_{sr}\tilde{\gamma}_0^3}{p_0} \left(1 + \frac{2}{\sqrt{1 + \frac{4\lambda_{sr}P_{s,1}}{\lambda_{sd}\lambda_{rd}\tilde{\gamma}_0}}} - 1 \right) \approx \frac{2\lambda_{sd}^2\lambda_{sr}\tilde{\gamma}_0^3}{p_0}, \quad (104)$$

in which the term $\frac{2}{\sqrt{1 + \frac{4\lambda_{sr}P_{s,1}}{\lambda_{sd}\lambda_{rd}\tilde{\gamma}_0}}} - 1$ in (104) is much smaller than 1 at high SNR, which can be ignored. Therefore,

the optimal transmission power $P_{s,1}$, $P_{s,2}$ and P_r are given by

$$P_{s,1} \approx \tilde{\gamma}_0 \left(\frac{2\lambda_{sd}^2\lambda_{sr}}{p_0} \right)^{\frac{1}{3}}, \quad (105)$$

$$P_{s,2} \approx \tilde{\gamma}_0 \left(\frac{\lambda_{sd}\lambda_{sr}^2}{2p_0^2} \right)^{\frac{1}{3}}, \quad (106)$$

$$P_r \approx \tilde{\gamma}_0 \left(\frac{\lambda_{sd}\lambda_{rd}}{2p_0} \right)^{\frac{1}{2}}. \quad (107)$$

From the above closed-form results, it is obvious that $(p_0^{-1})^{\frac{1}{3}} < (p_0^{-1})^{\frac{1}{2}} < (p_0^{-1})^{\frac{2}{3}}$ for a small targeted outage probability p_0 . Thus, we may expect that $P_{s,1} < P_r < P_{s,2}$. Under the assumption that $\frac{1}{\lambda_{sd}} = \frac{1}{\lambda_{sr}} = \frac{1}{\lambda_{rd}} = 1$ and $\mathcal{N}_0 = 1$, if we set $\gamma_0 = 10$ dB and $p_0 = 10^{-3}$, this expectation can be verified in Fig. 5, which shows that $P_{s,1} < P_r < P_{s,2}$.

Table 1: Comparison of the optimal power sequence by exhaustive search (82) and closed-form results (105)–(107) with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 2$ and dB scale.

Method	$P_{s,1}$	$P_{s,2}$	P_r
Exhaustive search (82)	21.16	29.04	23.42
(105)–(107)	21.00	29.00	23.50

3.1.6 Numerical and Simulation Results

In this section, we provide some numerical and simulation results to illustrate our theoretical development on the optimal power assignment strategy. In all studies, we assume that the variances of the channels h_{sd} , h_{sr} and h_{rd} are $\frac{1}{\lambda_{sd}} = \frac{1}{\lambda_{sr}} = \frac{1}{\lambda_{rd}} = 1$ and the noise variance is $\mathcal{N}_0 = 1$.

For comparison purposes, we derive the average total transmission power for the equal power assignment strategy. For a targeted outage probability p_0 , the equal power assignment approach also follows the constraint in (83) with equality as follows

$$\frac{\lambda_{sd}\tilde{\gamma}_0}{\Omega_L} \left(\frac{\lambda_{sr}\tilde{\gamma}_0}{\Omega_{L-1}} + \frac{\lambda_{rd}\tilde{\gamma}_0}{2(L-1)P_r} \right) = p_0. \quad (108)$$

Under the equal power assignment, the source and relay transmission powers are fixed over all (re)transmission rounds, i.e., $P_{s,l} = P_r = P$ for $1 \leq l \leq L$. Thus, the equal power assignment is

$$P = \tilde{\gamma}_0 \sqrt{\frac{\lambda_{sd}(2\lambda_{sr} + \lambda_{rd})}{2L(L-1)p_0}}. \quad (109)$$

By substituting (109) into (75), the approximation of the corresponding average total transmission power is given by

$$\begin{aligned} \bar{P}_{equ}(L) &\approx P + 2\lambda_{sd}\tilde{\gamma}_0 + \frac{\tilde{\gamma}_0^2}{P} \sum_{l=3}^L \frac{\lambda_{sd}(2\lambda_{sr} + \lambda_{rd})}{(l-1)(l-2)} \\ &= \tilde{\gamma}_0 \left[\sqrt{\frac{\lambda_{sd}(2\lambda_{sr} + \lambda_{rd})}{2L(L-1)p_0}} \{1 + 2L(L-2)p_0\} + 2\lambda_{sd} \right]. \end{aligned} \quad (110)$$

In Figs. 2, 3 and 4, we plot the average total transmission power of the cooperative H-ARQ relaying protocol by comparing the approximation result in (75), the exact closed-form result in (26) and the simulation result for the cases of $L = 2, 3$ and 4 , respectively. In this example, we assume that the source transmission power is fixed over all (re)transmission rounds and equal to the relay transmission power, i.e., $P_{s,l} = P_r = P$ for $1 \leq l \leq L$. The targeted SNR is set at $\gamma_0 = 10$ dB. Apparently, we can see that the closed-form result of the average total transmission power matches exactly with the simulation curve in each figure. We also observe that the approximation of the average total transmission power is loose at low SNR and tight at high SNR. The approximation curve is almost indistinguishable from the exact closed-form result and the simulation curve for SNR above 15 dB in each figure.

In Figs. 5–7, we compare the optimal transmission power sequence from Theorem 3 and the exhaustive search result based on the original optimization problem in (82) (without approximation). Note that in Fig. 5 ($L = 2$), we plot the optimal power sequence obtained from (105)–(107) instead of Theorem 3. In these three figures, we assumed that the targeted SNR is $\gamma_0 = 10$ dB and the required outage performance is $p_0 = 10^{-3}$.

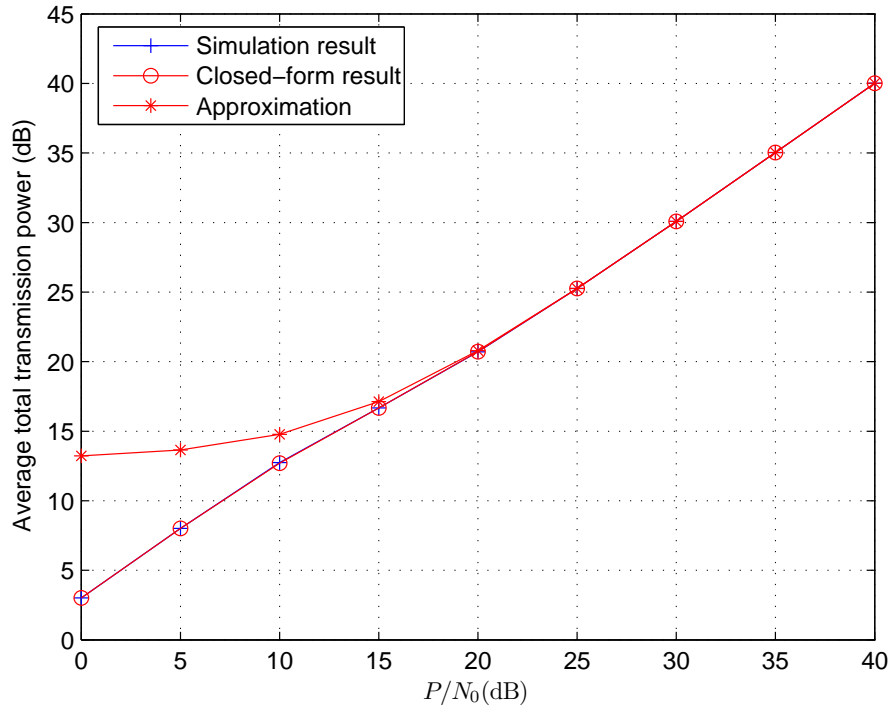


Figure 2: Average total power consumption per information packet with $P_{s,l} = P_r = P, 1 \leq l \leq 2$, $\gamma_0 = 10$ dB, $L = 2$.

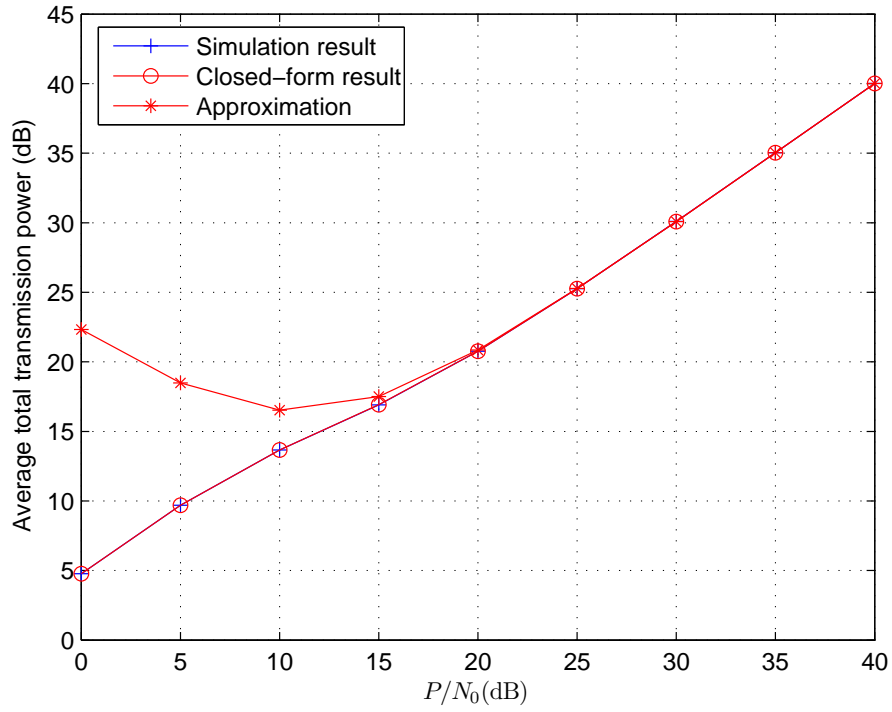


Figure 3: Average total power consumption per information packet with $P_{s,l} = P_r = P, 1 \leq l \leq 3$, $\gamma_0 = 10$ dB, $L = 3$.

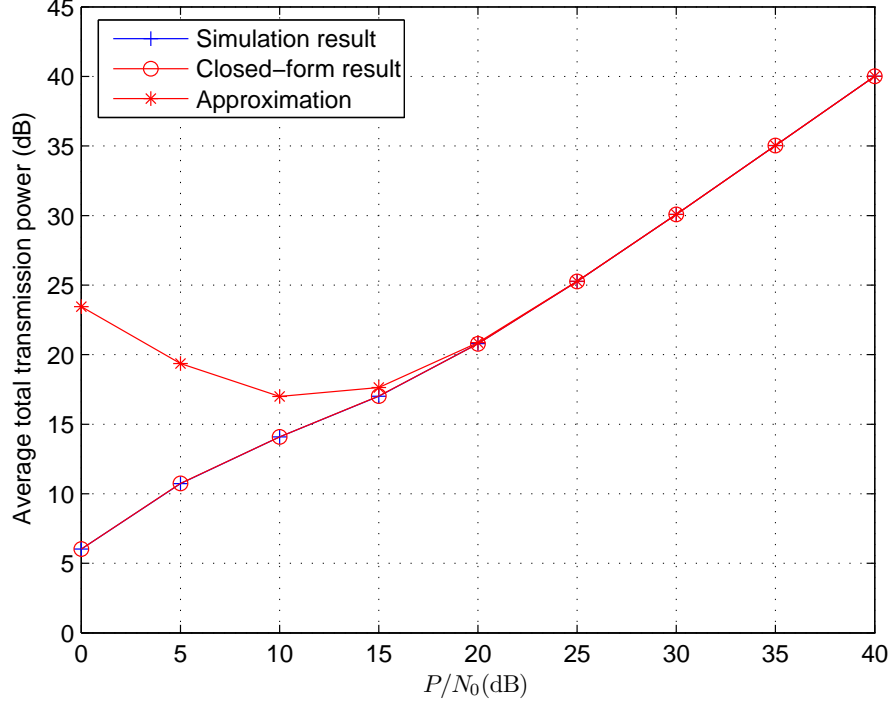


Figure 4: Average total power consumption per information packet with $P_{s,l} = P_r = P$, $1 \leq l \leq 4$, $\gamma_0 = 10$ dB, $L = 4$.

The maximum number of (re)transmission rounds is $L = 2$ in Fig. 5, $L = 3$ in Fig. 6 and $L = 4$ in Fig. 7. We can see that the optimal transmission power values resulted from Theorem 3 (solid line with ‘*’) match well with the exhaustive search result from the original optimization problem (solid line with ‘o’). Moreover, we present the comparison of the two solid lines numerically in Tables 1, 2 and 3 (with dB scale). When $L = 2$, Table 1 shows that the optimal power sequence from the closed-form results (105)–(107) is very close to the exhaustive search result. For comparison, we also include in the figures the transmission power level of the equal power assignment scheme (dashed line with ‘◊’). We observe that the optimal power assignment strategy assigns less transmission power to the source in the first few (re)transmission rounds and significantly large transmission power to the source at the last (L -th) round compared to the equal power assignment strategy. It provides some insightful information on how much power should be assigned to the source at each round for saving the average total power cost.

In Figs. 8, 9 and 10, we plot the average total transmission power for both the optimal power assignment scheme and the equal power assignment scheme, with different targeted SNR γ_0 (from 0 dB to 25 dB). The

Table 2: Comparison of the optimal power sequence by exhaustive search (82) and Theorem 3 with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 3$ and dB scale.

Method	$P_{s,1}$	$P_{s,2}$	$P_{s,3}$	P_r
Exhaustive search (82)	16.53	19.98	29.65	17.96
Theorem 3	17.20	20.13	29.58	17.63

Table 3: Comparison of the optimal power sequence by exhaustive search (82) and Theorem 3 with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 4$ and dB scale.

Method	$P_{s,1}$	$P_{s,2}$	$P_{s,3}$	$P_{s,4}$	P_r
Exhaustive search (82)	14.39	14.91	21.85	28.44	15.62
Theorem 3	15.54	15.48	21.65	28.30	15.05

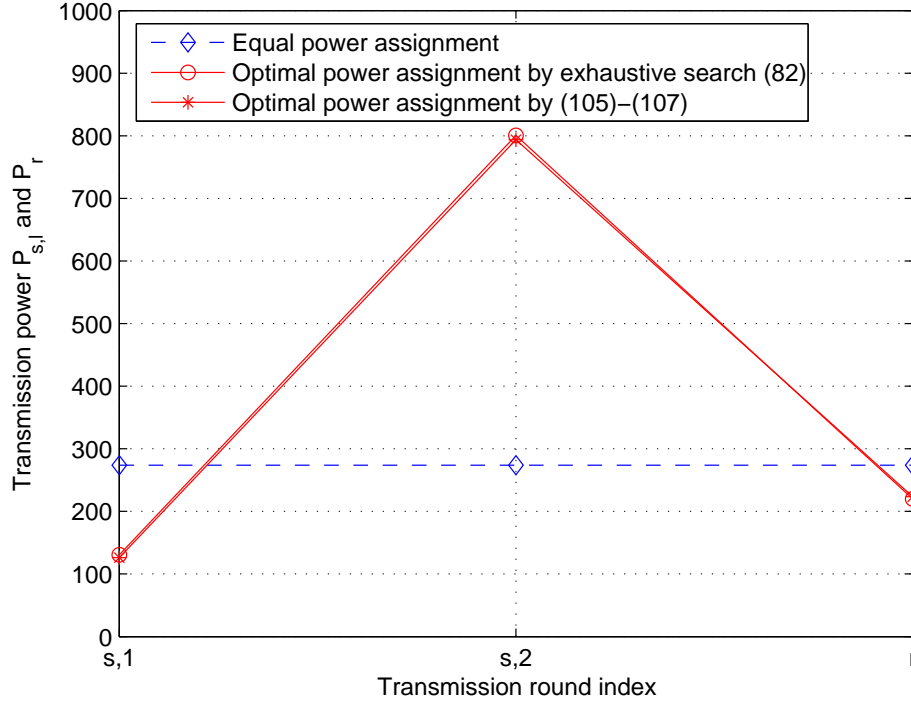


Figure 5: Transmission power sequence of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 2$.

required outage performance is set at $p_0 = 10^{-3}$. When $L = 2$, from Fig. 8 we observe that the optimal power assignment saves about 1.5 dB in average total transmission power compared to the equal power assignment. When $L = 3$ and 4, Figs. 9 and 10 show that the optimal power assignment scheme outperforms the equal power assignment scheme with a performance improvement of about 2.6 dB. Moreover, it is interesting to observe that in each figure, the performance gain of the optimal power assignment scheme is almost constant for different targeted SNR γ_0 (from 0 dB to 25 dB).

We also compare the average total transmission power required in the two power assignment strategies with different targeted outage probability values. We assume the required SNR is $\gamma_0 = 10$ dB. Figs. 11, 12 and 13 present comparison results for the cases of $L = 2, 3$ and 4, respectively. From the three figures, we can see that for an outage performance of $p_0 = 10^{-5}$, the power savings of the optimal power assignment compared to the equal power assignment are 5 dB when $L = 2$, 10 dB when $L = 3$, and 11 dB when $L = 4$. We also observe that the lower the required outage probability, the more performance gain of the optimal power assignment strategy compared to the equal power assignment strategy.

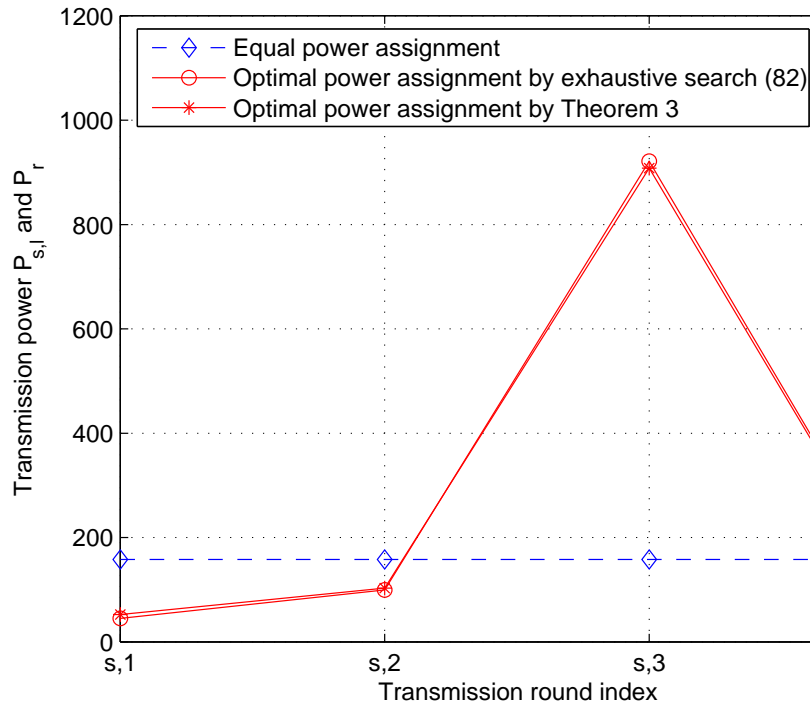


Figure 6: Transmission power sequence of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 3$.

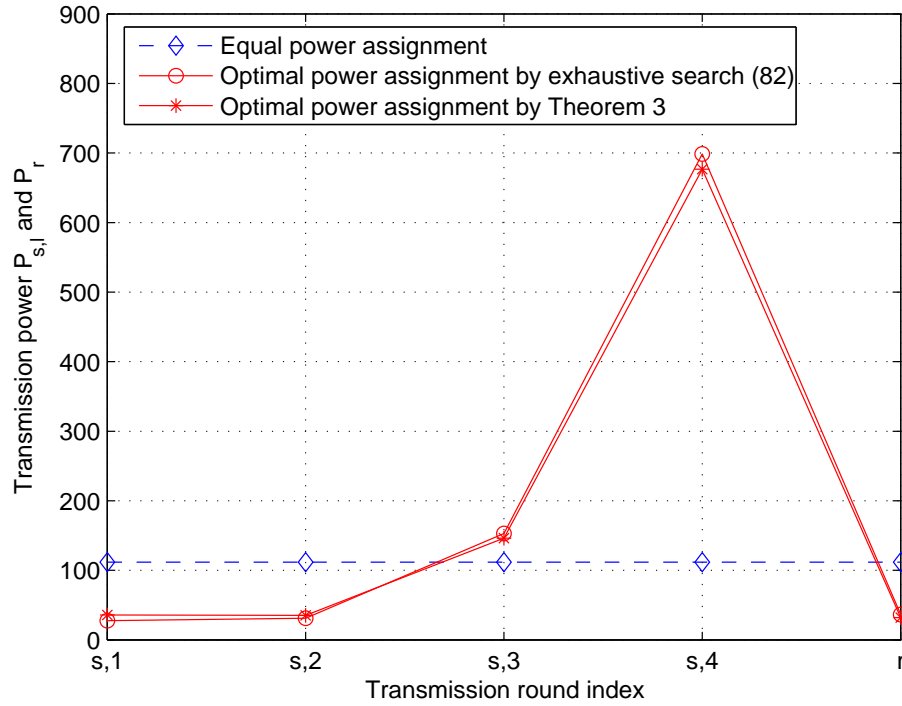


Figure 7: Transmission power sequence of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $p_0 = 10^{-3}$, $L = 4$.

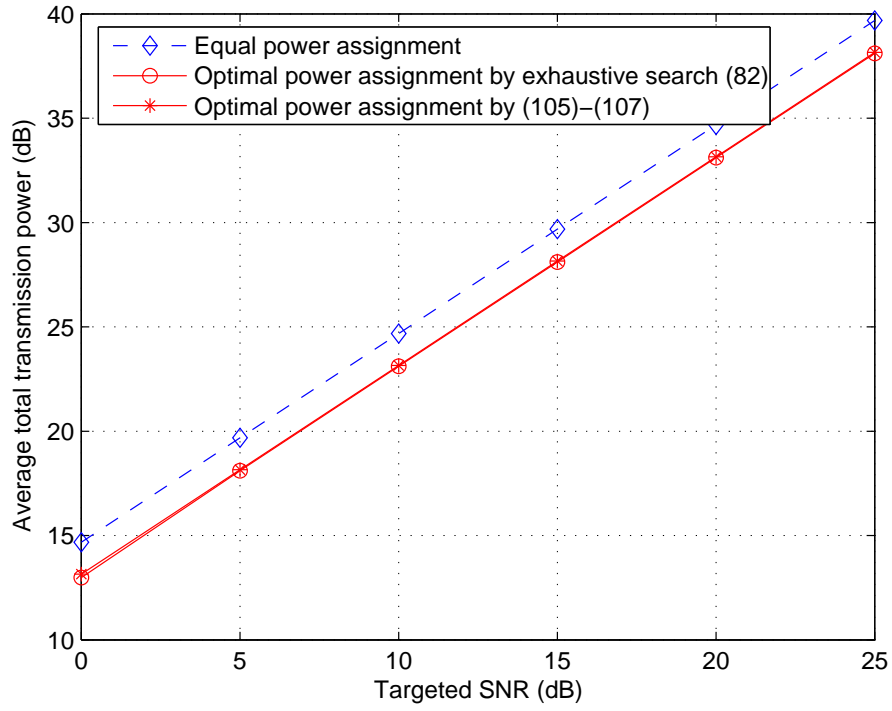


Figure 8: Average total transmission power of the optimal and equal power assignment strategies with $p_0 = 10^{-3}$, $L = 2$.

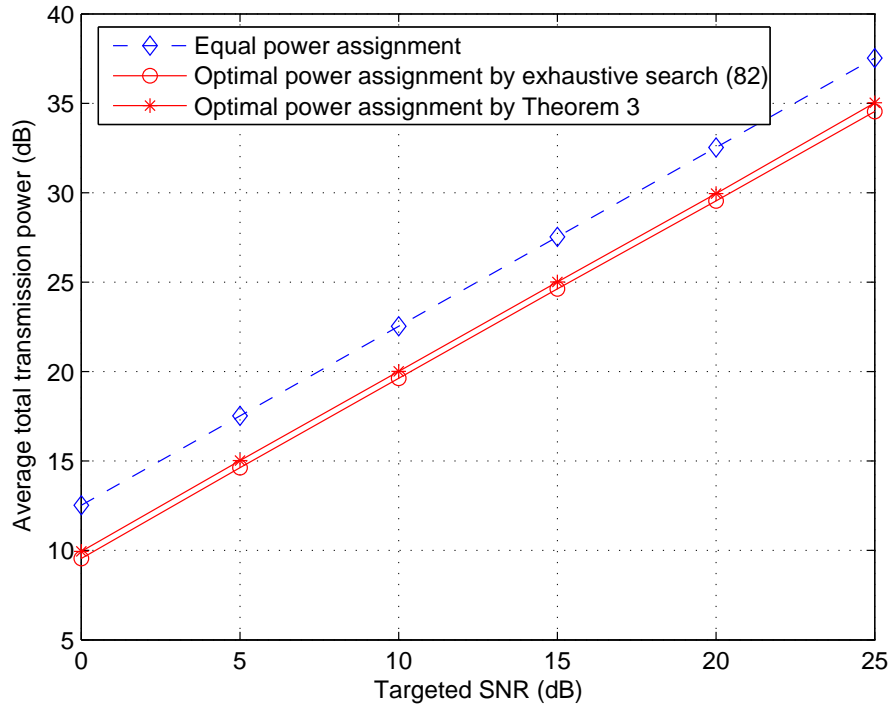


Figure 9: Average total transmission power of the optimal and equal power assignment strategies with $p_0 = 10^{-3}$, $L = 3$.

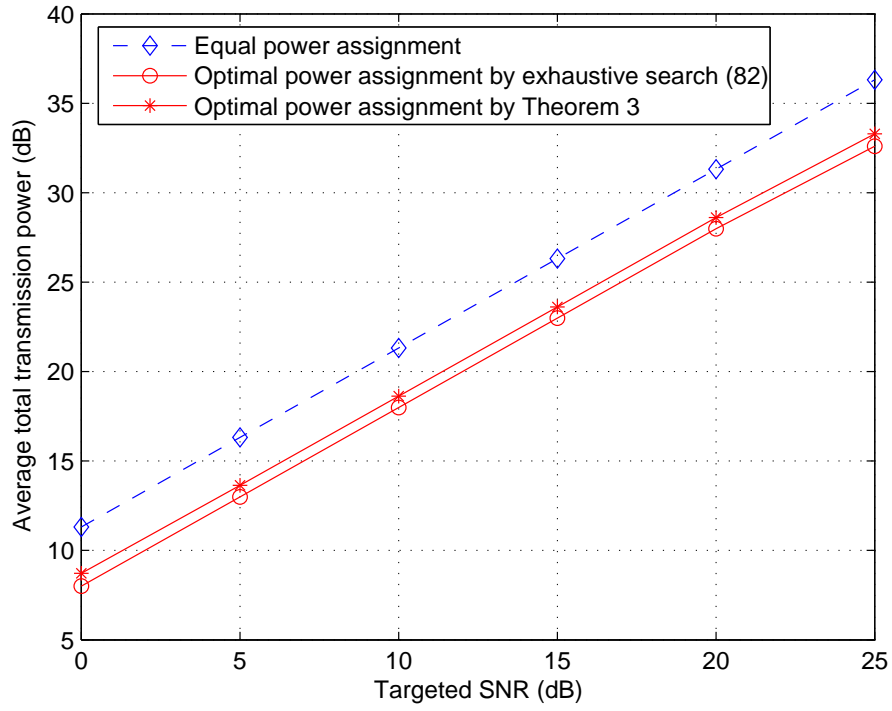


Figure 10: Average total transmission power of the optimal and equal power assignment strategies with $p_0 = 10^{-3}$, $L = 4$.

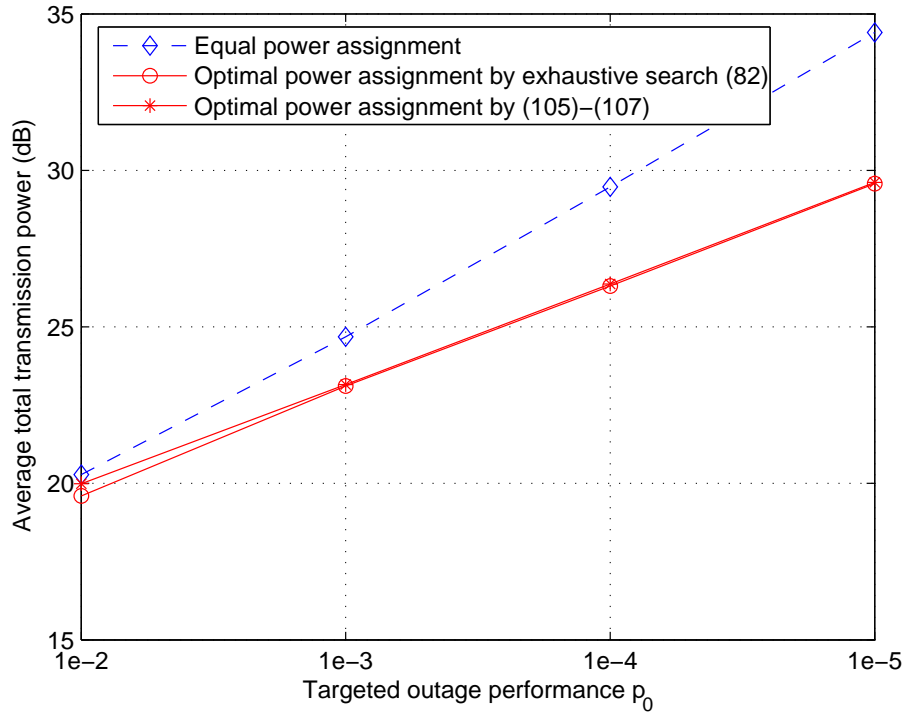


Figure 11: Average total transmission power of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $L = 2$.

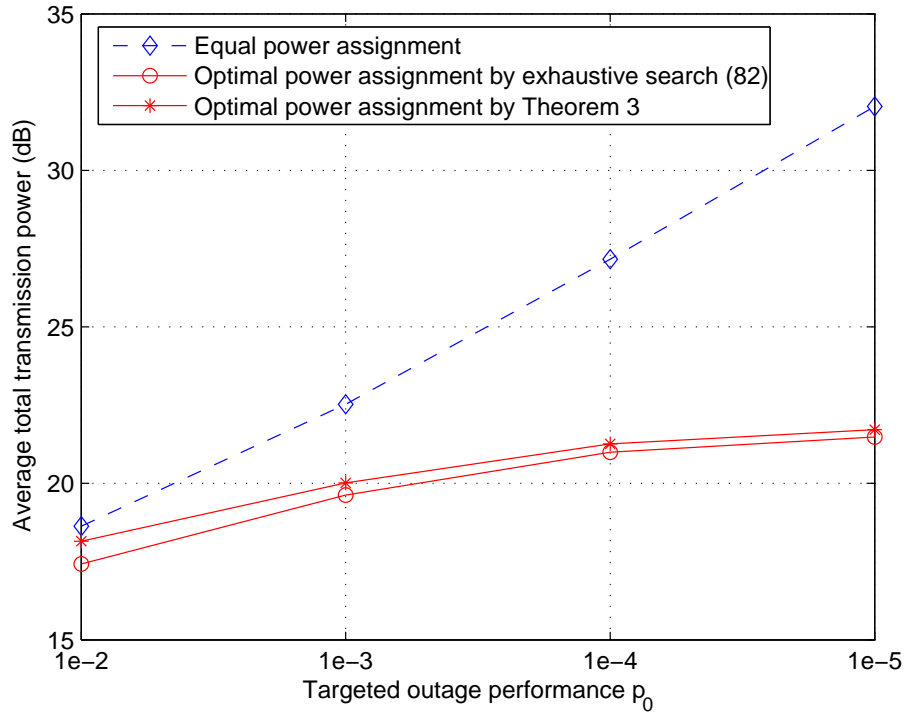


Figure 12: Average total transmission power of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $L = 3$.

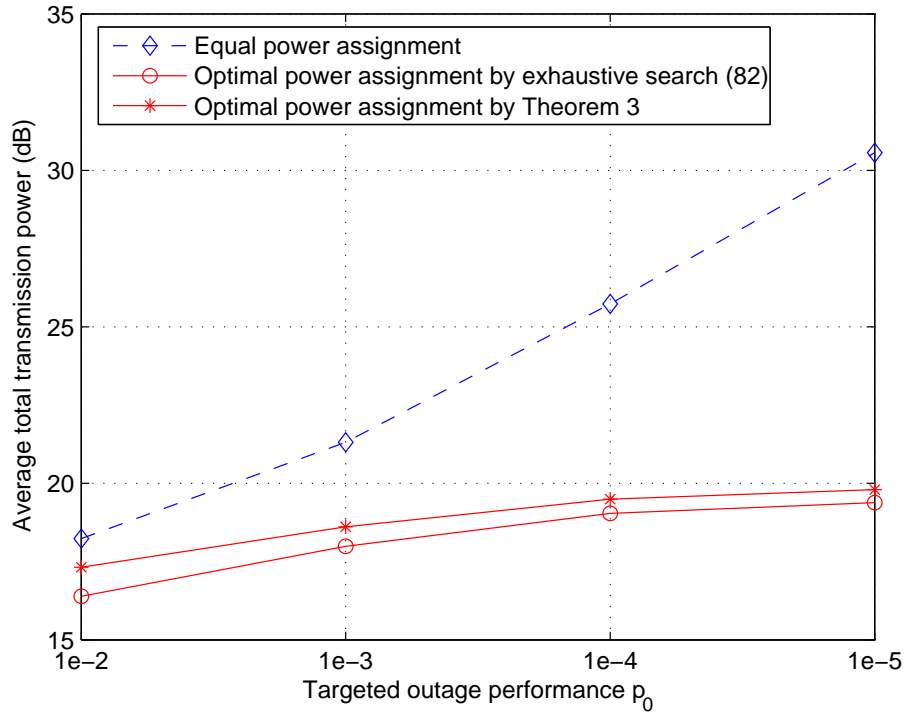


Figure 13: Average total transmission power of the optimal and equal power assignment strategies with $\gamma_0 = 10$ dB, $L = 4$.

Moreover, we show in Figs. 8–13 the average total transmission power from exhaustive search result based on the original optimization problem in (82). When $L = 2$, we can see from Fig. 8 that the average total transmission power from closed-form results (105)–(107) matches tightly with that from exhaustive search result. In Fig. 11, we also observe that the solid line with ‘*’ obtained from (105)–(107) is tight below the targeted outage probability of 10^{-3} . When $L = 3$ (Figs. 9 and 12) and $L = 4$ (Figs. 10 and 13), we can see that the average total transmission power from Theorem 3 is close to that from exhaustive search result, in which the gap between the two solid lines results from the fact that the solid line with ‘*’ is originated from the approximation result in (75) and the solid line with ‘o’ is obtained from the exact closed-form result in (26).

3.2 Cooperative Communication Protocol Designs Based on Optimum Power and Time Allocation

In this subsection, we design and optimize cooperative communication protocols by exploring all possible variations in time and power domains. First, we introduce a general system model for cooperative communication protocol design with flexibility in time/power allocation and arbitrary re-encoding methods. Second, we study the ideal cooperative protocol and determine the corresponding optimum time and power allocations analytically. Third, we design the practical cooperative communication protocol based on linear mapping and optimize the linear mapping function to minimize the outage probability of the protocol. Fourth, extensive numerical and simulation studies are presented to verify our theoretical development. We use the following notation throughout this subsection. Bold letters in uppercase and lowercase denote matrices and vectors, respectively. $(\cdot)^H$, $\det(\cdot)$ and $\text{Tr}(\cdot)$ represent Hermitian transpose, determinant and trace operators, respectively. \mathbf{I}_L is an $L \times L$ identity matrix, and $\mathbf{diag}(\lambda_1, \dots, \lambda_L)$ is an $L \times L$ diagonal matrix with diagonal elements $\lambda_1, \dots, \lambda_L$.

3.2.1 System Model

We consider a cooperative wireless network that consists of one source, one destination and one relay using DF relaying protocol. The cooperative strategy is described as followed with two phases. In Phase I, the source transmits information signal to the destination, and the signal is also received by the relay as well. In Phase II, if the relay is able to fully decode the information signal, it helps forwarding the information to the destination via certain re-encoding/transform methods. Throughout this paper, we consider narrowband transmissions in the wireless network in which channel between any two nodes is subject to the effects of frequency nonselective Rayleigh fading and additive white Gaussian noise (AWGN). We assume that the channel state information (CSI) is available only at the receivers, not at the transmitters. Nodes in the network work in a half-duplex mode where they cannot transmit and receive simultaneously in a same frequency band.

More specifically, in Phase I, the source broadcasts its information signal $x_s(t)$ to both the destination and the relay during time interval $(0, T_1]$. The received signals $y_{s,d}(t)$ and $y_{s,r}(t)$ at the destination and at the relay can be modeled as

$$y_{s,d}(t) = \sqrt{P_1} h_{s,d} x_s(t) + n_{s,d}(t), \quad 0 < t \leq T_1, \quad (111)$$

$$y_{s,r}(t) = \sqrt{P_1} h_{s,r} x_s(t) + n_{s,r}(t), \quad 0 < t \leq T_1, \quad (112)$$

where P_1 is the transmission power, $x_s(t)$ is normalized with power 1, and $n_{s,d}(t)$ and $n_{s,r}(t)$ are corresponding received noise at the destination and the relay. In (111) and (112), $h_{s,d}$ and $h_{s,r}$ are the channel coefficients from the source to the destination and the relay, respectively. In Phase II, if the relay fully decodes the information from the source, then the relay re-encodes the information and forwards it to the destination with power P_2 , otherwise the relay remains idle. We denote the time duration of Phase II as T_2 . Then, the received signal $y_{r,d}(t)$ at the destination in Phase II can be modeled as

$$y_{r,d}(t) = \sqrt{\tilde{P}_2} h_{r,d} x_r(t) + n_{r,d}(t), \quad T_1 < t \leq T_1 + T_2, \quad (113)$$

where $\tilde{P}_2 = P_2$ if the relay correctly decodes the information signal $x_s(t)$, otherwise $\tilde{P}_2 = 0$, $h_{r,d}$ is the channel coefficient from the relay to the destination, and $n_{r,d}(t)$ is received noise at the destination in Phase II. In (113),

$x_r(t) \triangleq \mathcal{M}(x_s(t))$ is a re-encoded version of the original information signal $x_s(t)$ and it is normalized with average power 1. We note that theoretically arbitrary re-encoding function $\mathcal{M}(\cdot)$ may be considered in the protocol design.

We assume that the channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variances $\delta_{s,d}^2$, $\delta_{s,r}^2$ and $\delta_{r,d}^2$, respectively. The noise terms $n_{s,d}(t)$, $n_{s,r}(t)$ and $n_{r,d}(t)$ are modeled as zero-mean AWGN with variance \mathcal{N}_0 . We denote the total time duration of each transmission period as $T \triangleq T_1 + T_2$ and assume that the fading channels are quasi-static within each transmission period. We denote the ratio of the time allocation in Phase I over the whole period as $\alpha \triangleq T_1/T \in (0, 1)$. If the average transmission power of the protocol in each transmission period is P , then the source and relay transmission powers P_1 and P_2 should satisfy the constraint

$$P_1 T_1 + P_2 T_2 = PT, \quad (114)$$

or equivalently $\alpha P_1 + (1 - \alpha) P_2 = P$. For convenience, we further denote β as the ratio of the energy consumed in Phase I over the total energy consumption in each transmission period, i.e., $\beta \triangleq \frac{P_1 T_1}{PT}$, so we have $\beta = \frac{\alpha P_1}{P}$.

We note that the system model specified above for cooperative relaying protocol designs has two variable domains to optimize: the power domain of allocating P_1 and P_2 and the time domain of allocating T_1 and T_2 . With given power budget P , we intend to optimize the power variables and the time variables such that the outage probability of the cooperative relaying protocol is minimized. First, we study an ideal cooperative relaying protocol where there is no constraint on the re-encoding function $\mathcal{M}(\cdot)$ and time allocation. The corresponding theoretical analysis will serve as guideline and benchmark in following practical cooperative communication protocol design. Then, with more realistic consideration for implementation, we propose a practical cooperative relaying protocol by considering linear mapping technique as the re-encoding function $\mathcal{M}(\cdot)$, and interestingly, the performance of the proposed practical protocol design is very close to the performance benchmark of the ideal cooperative protocol.

3.2.2 Ideal Cooperative Communication Protocol

In this section, we focus on joint optimization of power allocation and time allocation in an ideal cooperative protocol where the system can use arbitrary re-encoding function $\mathcal{M}(\cdot)$ at the relay and adjust time allocation arbitrarily between Phases I and II. It has been shown that the relay and the source can use independent codebooks to achieve maximum rate in cooperative communication protocols [46]. In the ideal cooperative protocol, we assume that the source and the relay perfectly cooperate by using two independent codebooks. We also assume that the system can arbitrarily allocate time duration to both phases, i.e., T_1 and T_2 can be arbitrary positive numbers ($T_1 + T_2 = T$). In the following, we first calculate the outage probability of the ideal cooperative protocol. Then, we obtain optimum power and time allocation to minimize the outage probability of the ideal cooperative protocol. For any give time allocation $\alpha \in (0, 1)$, we determine the optimum power allocation at the source and the relay analytically with closed-form expression. Finally, we show that in order to minimize the outage probability of the protocol, one should allocate more energy and time to Phase I than that to Phase II.

First, we would like to derive the outage probability of the ideal cooperative relaying protocol. Throughout the paper, the outage probability is defined as the probability that the maximum mutual information \mathcal{I} of the transceiver is smaller than a predetermined target transmission rate R_T . If $\mathcal{I} > R_T$, we assume that the receiver can decode the message correctly with negligible error probability.

In Phase I, with independently and identically distributed (i.i.d.) circularly symmetric complex Gaussian input signals, the maximum mutual information between the source and the destination is given by

$$\mathcal{I}_{s,d} = \alpha \log_2 \left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} \right), \quad (115)$$

in which the time allocation ratio α shows the fact that Phase I occupies time duration T_1 in each transmission period $(0, T]$. Since $h_{s,d} \sim \mathcal{CN}(0, \delta_{s,d}^2)$, then $|h_{s,d}|^2$ is an exponential random variable with parameter $\lambda_{s,d} =$

$1/\delta_{s,d}^2$. Thus, the probability that the destination decodes incorrectly in Phase I can be calculated as

$$Pr[\mathcal{I}_{s,d} < R_T] = 1 - \exp\left\{-\frac{\mathcal{N}_0}{P_1\delta_{s,d}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}. \quad (116)$$

Similarly, the maximum mutual information $\mathcal{I}_{s,r}$ between the source and the relay in Phase I is

$$\mathcal{I}_{s,r} = \alpha \log_2 \left(1 + \frac{P_1 |h_{s,r}|^2}{\mathcal{N}_0}\right), \quad (117)$$

and the outage probability that the relay fails to decode the message in Phase I is

$$Pr[\mathcal{I}_{s,r} < R_T] = 1 - \exp\left\{-\frac{\mathcal{N}_0}{P_1\delta_{s,r}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}. \quad (118)$$

If the relay fully decodes the message from the source, then the relay forwards the message to the destination by using an independent codebook. With two independent codebooks at the source and relay, the channels of source-destination and relay-destination can be viewed as a pair of parallel channels, thus the joint maximum mutual information from the source to the destination in the two phases is given by

$$\mathcal{I}_{joint} = \alpha \log_2 \left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0}\right) + (1 - \alpha) \log_2 \left(1 + \frac{P_2 |h_{r,d}|^2}{\mathcal{N}_0}\right). \quad (119)$$

Since $h_{r,d} \sim \mathcal{CN}(0, \delta_{r,d}^2)$, $|h_{r,d}|^2$ is an exponential random variable with parameter $\lambda_{r,d} = 1/\delta_{r,d}^2$. The probability density function (pdf) of the random variable $|h_{r,d}|^2$ is $f_{|h_{r,d}|^2}(z_{r,d}) = \frac{1}{\delta_{r,d}^2} \exp\left\{-\frac{z_{r,d}}{\delta_{r,d}^2}\right\}$ with $z_{r,d} \geq 0$. Similarly, the pdf of the random variable $|h_{s,d}|^2$ is given by $f_{|h_{s,d}|^2}(z_{s,d}) = \frac{1}{\delta_{s,d}^2} \exp\left\{-\frac{z_{s,d}}{\delta_{s,d}^2}\right\}$ with $z_{s,d} \geq 0$. Then, with the relay signal, the probability that the destination decodes incorrectly is given by

$$\begin{aligned} Pr[\mathcal{I}_{joint} < R_T] &= \int_0^{\frac{\mathcal{N}_0}{P_1}(2^{\frac{R_T}{\alpha}} - 1)} f_{|h_{s,d}|^2}(z_{s,d}) \left(\int_0^{\frac{\mathcal{N}_0}{P_2}(2^{\frac{R_T - \alpha \log_2(1 + P_1 z_{s,d}/\mathcal{N}_0)}{1 - \alpha}} - 1)} f_{|h_{r,d}|^2}(z_{r,d}) dz_{r,d} \right) dz_{s,d} \\ &= \int_0^{\frac{\mathcal{N}_0}{P_1}(2^{\frac{R_T}{\alpha}} - 1)} \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_2\delta_{r,d}^2}\left(2^{\frac{R_T - \alpha \log_2(1 + P_1 z_{s,d}/\mathcal{N}_0)}{1 - \alpha}} - 1\right)\right\} \right) \frac{\exp\left\{-\frac{z_{s,d}}{\delta_{s,d}^2}\right\}}{\delta_{s,d}^2} dz_{s,d} \end{aligned} \quad (120)$$

Note that the outage events of the cooperative relaying protocol have two possibilities: i) both the destination and the relay fail to decode the message in phase I; and ii) the destination fails to decode the message jointly in Phase II even when the relay fully decodes the message and helps forwarding the message to the destination. Therefore, the overall outage probability \mathcal{P}_{out} of the ideal cooperative protocol is given by

$$\begin{aligned} \mathcal{P}_{out} &= Pr[\mathcal{I}_{s,d} < R_T] Pr[\mathcal{I}_{s,r} < R_T] + (1 - Pr[\mathcal{I}_{s,r} < R_T]) Pr[\mathcal{I}_{joint} < R_T] \\ &= \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_1\delta_{s,d}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}\right) \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_1\delta_{s,r}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\}\right) + \exp\left\{-\frac{\mathcal{N}_0}{P_1\delta_{s,r}^2}\left(2^{\frac{R_T}{\alpha}} - 1\right)\right\} \\ &\quad \times \int_0^{\frac{\mathcal{N}_0}{P_1}(2^{\frac{R_T}{\alpha}} - 1)} \left(1 - \exp\left\{-\frac{\mathcal{N}_0}{P_2\delta_{r,d}^2}\left(2^{\frac{R_T - \alpha \log_2(1 + P_1 z_{s,d}/\mathcal{N}_0)}{1 - \alpha}} - 1\right)\right\}\right) \frac{1}{\delta_{s,d}^2} \exp\left\{-\frac{z_{s,d}}{\delta_{s,d}^2}\right\} dz_{s,d}. \end{aligned} \quad (121)$$

3.2.3 Optimum Power and Time Allocations

In this subsection, we would like to minimize the outage probability in (121) to determine optimum power and time allocations for the ideal cooperative relaying protocol. However, the closed-form expression in (121) is not tractable for analytical purpose. Therefore, in the following, we first derive an approximation of the outage probability (121) which is asymptotically tight, then we determine the optimum time and power allocations based on the asymptotically tight approximation of the outage probability.

We would like to use the first-order Taylor series approximation, i.e. $\exp(x) \approx 1 + x$ for x close to 0, to simplify the two terms $Pr[\mathcal{I}_{s,d} < R_T]$ and $Pr[\mathcal{I}_{s,r} < R_T]$ in (121). With high SNR where P_1/\mathcal{N}_0 is large, we have

$$Pr[\mathcal{I}_{s,d} < R_T] \approx \frac{\mathcal{N}_0(2^{\frac{R_T}{\alpha}} - 1)}{P_1\delta_{s,d}^2}, \quad (122)$$

$$Pr[\mathcal{I}_{s,r} < R_T] \approx \frac{\mathcal{N}_0(2^{\frac{R_T}{\alpha}} - 1)}{P_1\delta_{s,r}^2}. \quad (123)$$

Thus, the product of $Pr[\mathcal{I}_{s,d} < R_T]$ and $Pr[\mathcal{I}_{s,r} < R_T]$ can be approximated as

$$Pr[\mathcal{I}_{s,d} < R_T] \cdot Pr[\mathcal{I}_{s,r} < R_T] \approx \frac{\mathcal{N}_0^2}{P_1^2\delta_{s,d}^2} A(\alpha), \quad (124)$$

where

$$A(\alpha) = \frac{(2^{\frac{R_T}{\alpha}} - 1)^2}{\delta_{s,r}^2}. \quad (125)$$

Note that $A(\alpha) > 0$ and its first-order differential is continuous for any $\alpha \in (0, 1)$. Furthermore, we can see in (123) that the term $Pr[\mathcal{I}_{s,r} < R_T] \ll 1$ in the high SNR region, so we have $1 - Pr[\mathcal{I}_{s,r} < R_T] \approx 1$ with large P_1/\mathcal{N}_0 .

In order to obtain an asymptotically tight approximation for the term $Pr[\mathcal{I}_{joint} < R_T]$, we need the following lemma that was developed in [40].

Lemma 4 [40] Assume that u_{s_1} and v_{s_2} are two independent scalar random variables. If their cumulative distribution functions (CDF) satisfy the following properties:

$$\begin{aligned} \lim_{s_1 \rightarrow \infty} s_1 \cdot Pr[u_{s_1} < R] &= a \cdot f(R), \\ \lim_{s_2 \rightarrow \infty} s_2 \cdot Pr[v_{s_2} < R] &= b \cdot g(R), \end{aligned}$$

where a and b are constants, $f(R)$ and $g(R)$ are monotonically increasing functions, and the derivative of $g(R)$, denoted as $g'(R)$, is integrable, then the CDF of the sum of the two independent random variables has the following property:

$$\lim_{s_1, s_2 \rightarrow \infty} s_1 s_2 \cdot Pr[u_{s_1} + v_{s_2} < R] = ab \cdot \int_0^R f(r)g'(R-r)dr. \quad (126)$$

To use Lemma 4, we observe that \mathcal{I}_{joint} in (119) includes two independent random variables which can be written as $\mathcal{I}_{joint} = u_{s_1} + u_{s_2}$, where $u_{s_1} = \alpha \log_2 \left(1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0} \right)$ and $u_{s_2} = (1 - \alpha) \log_2 \left(1 + \frac{P_2|h_{r,d}|^2}{\mathcal{N}_0} \right)$. Let $s_1 = \frac{P_1}{\mathcal{N}_0}$ and $s_2 = \frac{P_2}{\mathcal{N}_0}$, and since $|h_{s,d}|^2$ and $|h_{r,d}|^2$ are exponential random variables with parameter $1/\delta_{s,d}^2$

and $1/\delta_{r,d}^2$, so we have

$$\lim_{s_1 \rightarrow \infty} s_1 \cdot \Pr[u_{s_1} < R_T] = \lim_{s_1 \rightarrow \infty} s_1 \cdot \Pr \left[|h_{s,d}|^2 < \frac{2^{\frac{R_T}{\alpha}} - 1}{s_1} \right] = \frac{1}{\delta_{s,d}^2} \underbrace{\left(2^{\frac{R_T}{\alpha}} - 1 \right)}_{\triangleq f(R_T)}, \quad (127)$$

$$\lim_{s_2 \rightarrow \infty} s_2 \cdot \Pr[u_{s_2} < R_T] = \lim_{s_2 \rightarrow \infty} s_2 \cdot \Pr \left[|h_{r,d}|^2 < \frac{2^{\frac{R_T}{1-\alpha}} - 1}{s_2} \right] = \frac{1}{\delta_{r,d}^2} \underbrace{\left(2^{\frac{R_T}{1-\alpha}} - 1 \right)}_{\triangleq g(R_T)}. \quad (128)$$

We can see that in (127) and (128), both $f(R_T)$ and $g(R_T)$ are monotonically increasing functions, and furthermore we have $g'(R_T) = \frac{\ln 2}{1-\alpha} 2^{\frac{R_T}{1-\alpha}}$. By applying Lemma 4, we obtain an approximation of $\Pr[\mathcal{I}_{joint} < R_T]$ at the high SNR region as

$$\begin{aligned} \Pr[\mathcal{I}_{joint} < R_T] &\approx \frac{\mathcal{N}_0^2}{P_1 P_2 \delta_{s,d}^2 \delta_{r,d}^2} \int_0^{R_T} f(r) g'(R_T - r) dr \\ &= \frac{\mathcal{N}_0^2}{P_1 P_2 \delta_{s,d}^2 \delta_{r,d}^2} \int_0^{R_T} \left(2^{\frac{r}{\alpha}} - 1 \right) \frac{\ln 2}{1-\alpha} 2^{\frac{R_T-r}{1-\alpha}} dr \\ &= \frac{\mathcal{N}_0^2}{P_1 P_2 \delta_{s,d}^2} \cdot \underbrace{\frac{\ln 2}{\delta_{r,d}^2 (1-\alpha)} 2^{\frac{R_T}{1-\alpha}} \int_0^{R_T} \left[2^{\frac{r(1-2\alpha)}{\alpha(1-\alpha)}} - 2^{\frac{r}{\alpha-1}} \right] dr}_{\triangleq B(\alpha)}. \end{aligned} \quad (129)$$

In (129), we observe that $B(\alpha) > 0$ and its first-order differential is continuous for any $\alpha \in (0, 1)$. To further simplify (129), we calculate the integral in $B(\alpha)$ by considering two cases: $\alpha = \frac{1}{2}$ and $\alpha \neq \frac{1}{2}$. When $\alpha = \frac{1}{2}$, $B(\alpha)$ can be derived as

$$B(\alpha) = \frac{2 \ln 2}{\delta_{r,d}^2} 2^{2R_T} \int_0^{R_T} [1 - 2^{-2r}] dr = \frac{1}{\delta_{r,d}^2} (2R_T \ln 2 \cdot 2^{2R_T} - 2^{2R_T} + 1). \quad (130)$$

When $\alpha \neq \frac{1}{2}$, since $\int_0^{R_T} 2^{cr} dr = \frac{1}{c \ln 2} [2^{cR_T} - 1]$ for any constant $c > 0$, we have

$$\begin{aligned} B(\alpha) &= \frac{\ln 2}{\delta_{r,d}^2 (1-\alpha)} 2^{\frac{R_T}{1-\alpha}} \left[\int_0^{R_T} 2^{\frac{r(1-2\alpha)}{\alpha(1-\alpha)}} dr - \int_0^{R_T} 2^{\frac{r}{\alpha-1}} dr \right] \\ &= \frac{\ln 2}{\delta_{r,d}^2 (1-\alpha)} 2^{\frac{R_T}{1-\alpha}} \left[\frac{\alpha(1-\alpha)}{(1-2\alpha) \ln 2} \left(2^{\frac{R_T(1-2\alpha)}{\alpha(1-\alpha)}} - 1 \right) - \frac{\alpha-1}{\ln 2} \left(2^{\frac{R_T}{\alpha-1}} - 1 \right) \right] \\ &= \frac{1}{\delta_{r,d}^2} \left(\frac{\alpha}{1-2\alpha} 2^{\frac{R_T}{\alpha}} + \frac{\alpha-1}{1-2\alpha} 2^{\frac{R_T}{1-\alpha}} + 1 \right). \end{aligned} \quad (131)$$

Therefore, $B(\alpha)$ in (129) can be given explicitly as

$$B(\alpha) = \begin{cases} \frac{1}{\delta_{r,d}^2} (2R_T \ln 2 \cdot 2^{2R_T} - 2^{2R_T} + 1), & \alpha = \frac{1}{2}; \\ \frac{1}{\delta_{r,d}^2} \left(\frac{\alpha}{1-2\alpha} 2^{\frac{R_T}{\alpha}} + \frac{\alpha-1}{1-2\alpha} 2^{\frac{R_T}{1-\alpha}} + 1 \right), & \text{otherwise.} \end{cases} \quad (132)$$

We summarize the above discussion on the approximation of the outage probability in the following theorem.

Theorem 4 *In the ideal cooperative protocol, the outage probability can be approximated as*

$$\mathcal{P}_{out} \approx \tilde{\mathcal{P}}_{out} \triangleq \frac{\mathcal{N}_0^2 A(\alpha)}{\delta_{s,d}^2 P_1^2} + \frac{\mathcal{N}_0^2 B(\alpha)}{\delta_{s,d}^2 P_1 P_2}, \quad (133)$$

where $A(\alpha)$ and $B(\alpha)$ are specified in (125) and (132), respectively, and the approximation is asymptotically tight at high SNR.

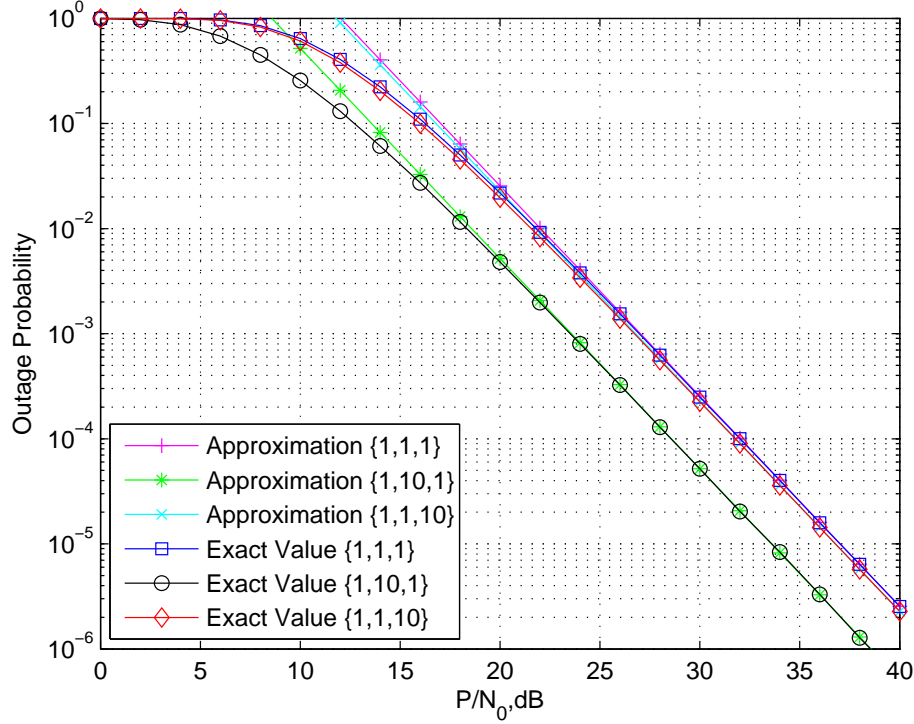


Figure 14: Comparison of the exact and approximation of the outage probability in three cases: $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1, 1\}$, $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 10, 1\}$ and $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1, 10\}$. We assume that $\alpha = \frac{1}{2}$, $R_T = 2$, $\mathcal{N}_0 = 1$, and $P_1 = P_2 = P$.

In Fig. 14, we compare the exact value of the outage probability which was calculated based on (121) and the asymptotic approximation of the outage probability in (133) in three scenarios with different channel variances: (i) $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1, 1\}$; (ii) $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 10, 1\}$; and (iii) $\{\delta_{s,d}^2, \delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1, 10\}$. In the comparison, we assume that $\alpha = \frac{1}{2}$, $R_T = 2$, $\mathcal{N}_0 = 1$, and $P_1 = P_2 = P$. From the figure, we can see that the approximation of the outage probability is tight at reasonable high SNR for various channel conditions. The curve of the outage probability approximation merges with the curve of the exact calculation at an outage probability of 10^{-2} in each case.

Next, we would like to jointly optimize power and time allocations for the ideal cooperative protocol based on the asymptotically tight approximation of the outage probability developed in Theorem 4. Note that the time allocation ratio $\alpha = T_1/T$ may take any number in the range of $(0, 1)$, while the power parameters P_1 and P_2 should satisfy the power constraint in (114), i.e. $\alpha P_1 + (1 - \alpha)P_2 = P$. The problem of optimizing time and power can be specified as follows:

$$\begin{aligned} \min_{\alpha, P_1, P_2} \tilde{\mathcal{P}}_{out}(\alpha, P_1, P_2) &\triangleq \frac{\mathcal{N}_0^2 A(\alpha)}{\delta_{s,d}^2 P_1^2} + \frac{\mathcal{N}_0^2 B(\alpha)}{\delta_{s,d}^2 P_1 P_2} \\ \text{s.t.} \quad &\alpha P_1 + (1 - \alpha)P_2 = P, \\ &0 < \alpha < 1, \quad P_1 > 0, \quad P_2 > 0. \end{aligned} \quad (134)$$

We found that for any given time allocation ratio $\alpha \in (0, 1)$, we are able to express the corresponding optimum powers P_1 and P_2 in terms of the time allocation ratio α with closed-form expressions, which are denoted as $P_1^*(\alpha)$ and $P_2^*(\alpha)$, respectively. Moreover, we found that for any time allocation ratio $\alpha \in (0, 1)$, the protocol should allocate more energy to Phase I than that to Phase II (i.e. $\beta = \frac{\alpha P_1^*(\alpha)}{P} > \frac{1}{2}$) in order to minimize the outage probability of the protocol. The results are summarized in the following theorem.

Theorem 5 In the ideal cooperative protocol, for any given time allocation ratio $\alpha \in (0, 1)$, the corresponding optimum powers $P_1^*(\alpha)$ and $P_2^*(\alpha)$ are given by:

$$P_1^*(\alpha) = \frac{1}{\alpha} \cdot \frac{1 + \sqrt{1 + 8[\alpha A(\alpha)]/[(1 - \alpha)B(\alpha)]}}{3 + \sqrt{1 + 8[\alpha A(\alpha)]/[(1 - \alpha)B(\alpha)]}} P, \quad (135)$$

$$P_2^*(\alpha) = \frac{1}{1 - \alpha} \cdot \frac{2}{3 + \sqrt{1 + 8[\alpha A(\alpha)]/[(1 - \alpha)B(\alpha)]}} P, \quad (136)$$

where $A(\alpha)$ and $B(\alpha)$ are specified in (125) and (132), respectively. Moreover, the resulting energy allocation ratio $\beta = \frac{\alpha P_1^*(\alpha)}{P}$ is strictly larger than $\frac{1}{2}$, which means that, the protocol should allocate more energy to Phase I than that to Phase II in order to minimize the outage probability.

Proof: For simplicity, in this proof we drop the parameter α in $A(\alpha)$ and $B(\alpha)$, and denote them as A and B , respectively. Based on the power constraint $\alpha P_1 + (1 - \alpha)P_2 = P$, the relay transmission power P_2 can be written as

$$P_2 = \frac{P - \alpha P_1}{1 - \alpha}. \quad (137)$$

Since $P_2 > 0$, (137) implies that $\alpha P_1 < P$. Therefore, for any given time allocation ratio $\alpha \in (0, 1)$, the problem (134) can be reduced as

$$\begin{aligned} \min_{P_1} \tilde{\mathcal{P}}_{out}(P_1) &= \frac{\mathcal{N}_0^2 A}{\delta_{s,d}^2 P_1^2} + \frac{\mathcal{N}_0^2 (1 - \alpha) B}{\delta_{s,d}^2 P_1 (P - \alpha P_1)} \\ \text{s.t. } &0 < \alpha P_1 < P. \end{aligned} \quad (138)$$

To find the optimal power P_1 in the problem (138), we proceed in two steps.

First, by taking derivative of the target function $\tilde{\mathcal{P}}_{out}(P_1)$ in terms of P_1 , we have

$$\frac{\partial \tilde{\mathcal{P}}_{out}(P_1)}{\partial P_1} = \frac{\mathcal{N}_0^2}{\delta_{s,d}^2} \frac{-2A(P - \alpha P_1)^2 - (1 - \alpha)B(P - 2\alpha P_1)P_1}{P_1^3 (P - \alpha P_1)^2}. \quad (139)$$

Let $\frac{\partial \tilde{\mathcal{P}}_{out}(P_1)}{\partial P_1} = 0$, we have

$$-2A(P - \alpha P_1)^2 - (1 - \alpha)B(P - 2\alpha P_1)P_1 = 0. \quad (140)$$

By solving the above equation, we have two possible solutions as follows:

$$P_{1,\pm}^* = \frac{[4\alpha A - (1 - \alpha)B] \pm \sqrt{8\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{4\alpha[\alpha A - (1 - \alpha)B]} P. \quad (141)$$

Second, for the two possible solutions, we would like to show that only the solution $P_{1,-}^*$ can satisfy the constraint in (138). We detail the discussion in two scenarios:

1) When $\alpha A > (1 - \alpha)B$, in this case the denominators of $P_{1,-}^*$ and $P_{1,+}^*$ are positive. We have

$$\begin{aligned}
P_{1,-}^* : \quad \alpha P_{1,-}^* &= \frac{[4\alpha A - (1 - \alpha)B] - \sqrt{8\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{4[\alpha A - (1 - \alpha)B]} P \\
&> \frac{[4\alpha A - (1 - \alpha)B] - \sqrt{4\alpha^2 A^2 + 4\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{4[\alpha A - (1 - \alpha)B]} P \\
&> \frac{[4\alpha A - (1 - \alpha)B] - \sqrt{[2\alpha A + (1 - \alpha)B]^2}}{4[\alpha A - (1 - \alpha)B]} P = \frac{P}{2}, \\
\alpha P_{1,-}^* &< \frac{[4\alpha A - (1 - \alpha)B] - \sqrt{9(1 - \alpha)^2 B^2}}{4[\alpha A - (1 - \alpha)B]} P = P; \\
P_{1,+}^* : \quad \alpha P_{1,+}^* &= \frac{[4\alpha A - (1 - \alpha)B] + \sqrt{8\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{4[\alpha A - (1 - \alpha)B]} P \\
&> \frac{[4\alpha A - (1 - \alpha)B] - \sqrt{9(1 - \alpha)^2 B^2}}{4[\alpha A - (1 - \alpha)B]} P > P.
\end{aligned}$$

We can see that the solution $P_{1,+}^*$ does not satisfy the power constraint and only $P_{1,-}^*$ satisfies the power constraint in this case. Moreover, the corresponding energy allocation ratio $\beta = \frac{\alpha P_{1,-}^*}{P}$ is within $(\frac{1}{2}, 1)$.

2) When $\alpha A < (1 - \alpha)B$, in this case the denominators of $P_{1,-}^*$ and $P_{1,+}^*$ in (141) are negative. For convenience, we multiply both their denominators and numerators by -1 , then we have

$$\begin{aligned}
P_{1,-}^* : \quad \alpha P_{1,-}^* &= \frac{-[4\alpha A - (1 - \alpha)B] + \sqrt{8\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{-4[\alpha A - (1 - \alpha)B]} P \\
&> \frac{-[4\alpha A - (1 - \alpha)B] + \sqrt{4\alpha^2 A^2 + 4\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{-4[\alpha A - (1 - \alpha)B]} P \\
&> \frac{-[4\alpha A - (1 - \alpha)B] + \sqrt{[2\alpha A + (1 - \alpha)B]^2}}{-4[\alpha A - (1 - \alpha)B]} P = \frac{P}{2}, \\
\alpha P_{1,-}^* &< \frac{-[4\alpha A - (1 - \alpha)B] + \sqrt{9(1 - \alpha)^2 B^2}}{-4[\alpha A - (1 - \alpha)B]} P = P; \\
P_{1,+}^* : \quad \alpha P_{1,+}^* &= \frac{-[4\alpha A - (1 - \alpha)B] - \sqrt{8\alpha(1 - \alpha)AB + (1 - \alpha)^2 B^2}}{-4[\alpha A - (1 - \alpha)B]} P \\
&< \frac{-[4\alpha A - (1 - \alpha)B] - (1 - \alpha)B}{-4[\alpha A - (1 - \alpha)B]} P < 0.
\end{aligned}$$

We can see that the solution $P_{1,-}^*$ is the only feasible solution satisfying the power constraint in (138), and the corresponding energy allocation ratio β is within the interval $(\frac{1}{2}, 1)$.

The above discussion shows that $P_{1,-}^*$ is the only solution that satisfies the power constraint and minimize the outage probability. The resulting energy allocation ratio β is always strictly larger than $\frac{1}{2}$.

We may rewrite the optimum power $P_{1,-}^*$ as

$$P_{1,-}^* = \frac{1 + \sqrt{1 + 8(\alpha A)/[(1 - \alpha)B]}}{3 + \sqrt{1 + 8(\alpha A)/[(1 - \alpha)B]}} \frac{P}{\alpha}. \quad (142)$$

Note that A and B are shorthands of $A(\alpha)$ and $B(\alpha)$, respectively. For completeness, for any given time allocation ratio $\alpha \in (0, 1)$, the optimum source transmission power $P_1^*(\alpha)$ is given by

$$P_1^*(\alpha) = \frac{1 + \sqrt{1 + 8[\alpha A(\alpha)]/[(1 - \alpha)B(\alpha)]}}{3 + \sqrt{1 + 8[\alpha A(\alpha)]/[(1 - \alpha)B(\alpha)]}} \frac{P}{\alpha}. \quad (143)$$

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Based on (137), the corresponding optimum relay transmission power $P_2^*(\alpha)$ is given by

$$P_2^*(\alpha) = \frac{2}{3 + \sqrt{1 + 8[\alpha A(\alpha)]/[(1 - \alpha)B(\alpha)]}} \frac{P}{1 - \alpha}. \quad (144)$$

Therefore, we prove Theorem 5 completely. \square

Based on Theorem 5, we substitute the optimum power solutions $P_1^*(\alpha)$ and $P_2^*(\alpha)$ into the optimization problem (134) to find the optimum time allocation ratio, i.e.

$$\begin{aligned} \min_{\alpha} \tilde{\mathcal{P}}_{out}(\alpha) &\triangleq \frac{\mathcal{N}_0^2 A(\alpha)}{\delta_{s,d}^2 P_1^*(\alpha)^2} + \frac{\mathcal{N}_0^2 B(\alpha)}{\delta_{s,d}^2 P_1^*(\alpha) P_2^*(\alpha)} \\ s.t. \quad &0 < \alpha < 1. \end{aligned} \quad (145)$$

We can see that the optimization in (145) has only a single variable α , and we can apply numerical search of the single variable α over the interval $(0, 1)$ to obtain the optimum time allocation ratio α^* that minimizes the asymptotic outage probability $\tilde{\mathcal{P}}_{out}$. With the optimum time allocation ratio α^* , based on Theorem 5 we can get the corresponding optimum source transmission power $P_1^*(\alpha^*)$ and the optimum relay transmission power $P_2^*(\alpha^*)$. Let us denote $(\alpha^*, P_1^*(\alpha^*), P_2^*(\alpha^*))$ as the solution of optimum power and time allocation for the ideal cooperative protocol. We have the following result regarding the optimum time allocation α^* .

Theorem 6 *In the ideal cooperative protocol, the optimum time allocation ratio α^* is strictly larger than $\frac{1}{2}$, i.e. $\alpha^* \in (\frac{1}{2}, 1)$, which means that the protocol should allocate more time to Phase I than to Phase II in order to minimize the outage probability of the protocol.*

Proof: We would like to prove the result by contradiction. If there exists an optimum solution (α^*, P_1^*, P_2^*) with $\alpha^* \leq \frac{1}{2}$ that achieves the minimum outage probability $\tilde{\mathcal{P}}_{out}^*$ in the problem (134), we will find another solution that results in smaller outage probability to contradict the assumption.

With the assumption of the optimum solution (α^*, P_1^*, P_2^*) with $\alpha^* \leq \frac{1}{2}$, the resulting minimum outage probability $\tilde{\mathcal{P}}_{out}^*$ can be expressed as

$$\tilde{\mathcal{P}}_{out}^* = \tilde{\mathcal{P}}_{out}(\alpha^*, P_1^*, P_2^*) = \frac{\mathcal{N}_0^2 A(\alpha^*)}{\delta_{s,d}^2 (P_1^*)^2} + \frac{\mathcal{N}_0^2 B(\alpha^*)}{\delta_{s,d}^2 P_1^* P_2^*}. \quad (146)$$

Let us consider a new family of resource allocation strategy $(\hat{\alpha}, \hat{P}_1, \hat{P}_2) = (\tau\alpha^*, \frac{P_1^*}{\tau}, \frac{(1-\alpha^*)P_2^*}{1-\tau\alpha^*})$ for $0 < \tau < \frac{1}{\alpha^*}$. We can check that the new resource allocation strategy $(\hat{\alpha}, \hat{P}_1, \hat{P}_2)$ satisfies the power constraint in (134). Especially when $\tau = 1$, the new resource allocation solution is reduced to the optimum solution (α^*, P_1^*, P_2^*) . With the new resource allocation strategy, the resulting outage probability is

$$C(\tau) \triangleq \tilde{\mathcal{P}}_{out}(\hat{\alpha}, \hat{P}_1, \hat{P}_2) = \frac{\mathcal{N}_0^2 (\tau\alpha^*)^2 A(\tau\alpha^*)}{\delta_{s,d}^2 (P_1^* \alpha^*)^2} + \frac{\mathcal{N}_0^2 \tau\alpha^* (1 - \tau\alpha^*) B(\tau\alpha^*)}{\delta_{s,d}^2 P_1^* P_2^* \alpha^* (1 - \alpha^*)}. \quad (147)$$

Since $\frac{\ln 2}{\alpha} \int_0^{R_T} 2^{\frac{v}{\alpha}} dv = 2^{\frac{R_T}{\alpha}} - 1$, we can rewrite the functions $A(\alpha)$ and $B(\alpha)$, defined in (125) and (129) respectively, as follows:

$$A(\alpha) = \frac{(\ln 2)^2}{\delta_{s,r}^2 \alpha^2} \int_0^{R_T} \int_0^{R_T} 2^{\frac{v}{\alpha}} 2^{\frac{r}{\alpha}} dv dr, \quad (148)$$

$$B(\alpha) = \frac{(\ln 2)^2}{\delta_{r,d}^2 \alpha (1 - \alpha)} \int_0^{R_T} \int_0^{R_T - r} 2^{\frac{v}{\alpha}} 2^{\frac{r}{1 - \alpha}} dv dr. \quad (149)$$

Thus, $C(\tau)$ can be written as

$$C(\tau) = c_1 \int_0^{R_T} \int_0^{R_T} 2^{\frac{v}{\tau\alpha^*}} 2^{\frac{r}{\tau\alpha^*}} dv dr + c_2 \int_0^{R_T} \int_0^{R_T - r} 2^{\frac{v}{\tau\alpha^*}} 2^{\frac{r}{1 - \tau\alpha^*}} dv dr, \quad (150)$$

where $c_1 = \frac{\mathcal{N}_0^2(\ln 2)^2}{\delta_{s,d}^2 \delta_{s,r}^2 (P_1^* \alpha^*)^2}$ and $c_2 = \frac{\mathcal{N}_0^2(\ln 2)^2}{\delta_{s,d}^2 \delta_{r,d}^2 P_1^* P_2^* \alpha^* (1-\alpha^*)}$ are positive constants.

In the following, we would like to show that the differential of the function $C(\tau)$ at $\tau = 1$ is negative, i.e., $\frac{\partial C(\tau)}{\partial \tau} \Big|_{\tau=1} < 0$. By taking derivative over the function $C(\tau)$, we have

$$\begin{aligned} \frac{\partial C(\tau)}{\partial \tau} \Big|_{\tau=1} &= c_1 \ln 2 \int_0^{R_T} \int_0^{R_T} \left(-\frac{v+r}{\alpha^*} 2^{\frac{v+r}{\alpha^*}} \right) dvdr \\ &\quad + c_2 \alpha^* \ln 2 \int_0^{R_T} \int_0^{R_T-r} 2^{\frac{v}{\alpha^*}} 2^{\frac{r}{1-\alpha^*}} \left(-\frac{v}{(\alpha^*)^2} + \frac{r}{(1-\alpha^*)^2} \right) dvdr. \end{aligned} \quad (151)$$

We can see that in (151), the first term is strictly less than 0. Next, we would like to show that the second term in (151) is non-positive. We denote the integrand as $b(v, r; \alpha^*) = 2^{\frac{v}{\alpha^*}} 2^{\frac{r}{1-\alpha^*}} \left(-\frac{v}{(\alpha^*)^2} + \frac{r}{(1-\alpha^*)^2} \right)$ and the corresponding integration domain as $\Delta = \{(v, r) \in \mathbb{R}_+^2 : v + r < R_T\}$, then we can rewrite the second term of (151) as $c_2 \alpha^* \ln 2 \iint_{\Delta} b(v, r; \alpha^*) dvdr$. The symmetric property of the integration domain Δ implies that if $(v, r) \in \Delta$, then $(r, v) \in \Delta$ as well. Thus, the second term of (151) can be given by

$$c_2 \alpha^* \ln 2 \iint_{\Delta} b(v, r; \alpha^*) dvdr = \frac{c_2 \alpha^* \ln 2}{2} \iint_{\Delta} [b(v, r; \alpha^*) + b(r, v; \alpha^*)] dvdr. \quad (152)$$

To prove the second term of (151) is non-positive, it is sufficient to prove that the sum of $b(v, r; \alpha^*)$ and $b(r, v; \alpha^*)$ is non-positive for any $(v, r) \in \Delta$. Since $\alpha^* \leq \frac{1}{2}$, we can see that only if $v < r$, we may have $b(v, r; \alpha^*) > 0$, which means that $b(v, r; \alpha^*)$ and $b(r, v; \alpha^*)$ cannot be positive simultaneously. So, there are only two possible cases to consider. When both $b(v, r; \alpha^*)$ and $b(r, v; \alpha^*)$ are non-positive, the proof is trivial. When either $b(v, r; \alpha^*)$ or $b(r, v; \alpha^*)$ is positive, without loss of generality, we assume that $b(v, r; \alpha^*) > 0$ and $b(r, v; \alpha^*) \leq 0$, then we have

$$\begin{aligned} b(v, r; \alpha^*) + b(r, v; \alpha^*) &= 2^{\frac{v}{\alpha^*}} 2^{\frac{r}{1-\alpha^*}} \left(-\frac{v}{(\alpha^*)^2} + \frac{r}{(1-\alpha^*)^2} \right) + 2^{\frac{r}{\alpha^*}} 2^{\frac{v}{1-\alpha^*}} \left(-\frac{r}{(\alpha^*)^2} + \frac{v}{(1-\alpha^*)^2} \right) \\ &\leq 2^{\frac{r}{\alpha^*}} 2^{\frac{v}{1-\alpha^*}} \left(-\frac{r+v}{(\alpha^*)^2} + \frac{r+v}{(1-\alpha^*)^2} \right) \leq 0, \end{aligned} \quad (153)$$

where the first inequality is due to the fact that $v < r$. The above result implies that the second term of (151) is non-positive. Therefore, we conclude that $\frac{\partial C(\tau)}{\partial \tau} \Big|_{\tau=1} < 0$.

The result of $\frac{\partial C(\tau)}{\partial \tau} \Big|_{\tau=1} < 0$ implies that we are able to find a τ ($\tau > 1$) such that $C(\tau) < C(1)$. Since $C(1)$ is the outage probability with the optimum allocation strategy (α^*, P_1^*, P_2^*) and $C(\tau)$ is the outage probability with another feasible allocations strategy $(\tau \alpha^*, \frac{P_1^*}{\tau}, \frac{(1-\alpha^*)P_2^*}{1-\tau \alpha^*})$, the fact that $C(\tau) < C(1)$ for some $\tau > 1$ contradicts the assumption that there exists an optimum solution (α^*, P_1^*, P_2^*) with $\alpha^* \leq \frac{1}{2}$. Therefore, we prove Theorem 6 completely. \square

Theorem 6 shows that the equal-time allocation strategy in most existing cooperative relaying protocol (allocating equal time among the two phases) is not optimal in general.

3.2.4 Practical Cooperative Communication Protocol Design Based on Linear Mapping

It is difficult, sometimes may be infeasible, to implement the ideal cooperative protocol as the re-encoding function $\mathcal{M}(\cdot)$ and time allocation can be arbitrary. In this section, with more realistic consideration, we design a cooperative communication protocol based on linear mapping, where the relay uses linear mapping as the re-encoding function $\mathcal{M}(\cdot)$. Also, the practical cooperative protocol allocates integer time slots in Phases I and II. It is much easier to implement the linear mapping forwarding method with the time allocation of integer time slots in both phases. The theoretical results from the ideal cooperative protocol in the previous section will serve as guideline and benchmark for the proposed linear-mapping based cooperative protocol design. Interestingly, the practical cooperative protocol based on optimum linear mapping performs closely to the performance benchmark of the ideal cooperative protocol. In the following, we first specify the practical

cooperative communication protocol design based on linear mapping. Then we optimize the linear mapping of the protocol such that the outage probability of the proposed protocol is minimized.

We intend to design a practical cooperative relaying protocol with L time slots in Phase I and K time slots in Phase II. We assume that each time slot has time duration T_s . In Phase I, the source broadcasts a block of symbols \mathbf{x}_s ($\mathbf{x}_s = (x_s[1], \dots, x_s[L])^T$) using L time slots, in which each element of \mathbf{x}_s has unit power. In Phase II, if the relay successfully decodes the message, then the relay takes K time slots to forward a re-encoded message \mathbf{x}_r ($\mathbf{x}_r = (x_r[1], \dots, x_r[K])^T$), where $\mathbf{x}_r = \mathbf{G}\mathbf{x}_s$ and \mathbf{G} is a $K \times L$ matrix representation of the linear mapping. In the protocol, the total time duration of Phase I is $T_1 = LT_s$ and the time duration of Phase II is $T_2 = KT_s$, thus the time allocation ratio α of the protocol is

$$\alpha = \frac{LT_s}{LT_s + KT_s} = \frac{L}{L + K}. \quad (154)$$

Based on the theoretical results in the previous section, we know that we should allocate more time to Phase I than that to Phase II in order to minimize the outage probability (i.e., $\frac{1}{2} < \alpha < 1$), which means that we should choose $K < L$ in the practical cooperative protocol design.

The general system model in Section 3.2.1 can be modified for the linear-mapping based cooperative protocol in a discrete version as follows:

$$\mathbf{y}_{s,d} = \sqrt{P_1} \mathbf{H}_{s,d} \mathbf{x}_s + \mathbf{n}_{s,d}, \quad (155)$$

$$\mathbf{y}_{s,r} = \sqrt{P_1} \mathbf{H}_{s,r} \mathbf{x}_s + \mathbf{n}_{s,r}, \quad (156)$$

$$\mathbf{y}_{r,d} = \sqrt{\tilde{P}_2} \mathbf{H}_{r,d} \mathbf{x}_r + \mathbf{n}_{r,d}, \quad (157)$$

where $\mathbf{H}_{s,d} = h_{s,d} \mathbf{I}_L$, $\mathbf{H}_{s,r} = h_{s,r} \mathbf{I}_L$, $\mathbf{H}_{r,d} = h_{r,d} \mathbf{I}_K$, $\mathbf{y}_{s,d}$ and $\mathbf{y}_{s,r}$ are the signal vectors of size $L \times 1$ received at the destination and the relay respectively during Phase I, $\mathbf{y}_{r,d}$ is the signal vector of size $K \times 1$ received at the destination during Phase II, $\mathbf{n}_{s,d}$ and $\mathbf{n}_{s,r}$ are the noise vectors of size $L \times 1$ at the destination and the relay respectively in Phase I, and $\mathbf{n}_{r,d}$ is the noise vector of size $K \times 1$ at the destination in Phase II. In (155)–(157), the elements of the noise vectors $\mathbf{n}_{s,d}$, $\mathbf{n}_{s,r}$ and $\mathbf{n}_{r,d}$ are independent Gaussian random variables with mean zero and variance \mathcal{N}_0 . The channel coefficients $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are modeled as zero-mean complex Gaussian random variables with variances $\delta_{s,d}^2$, $\delta_{s,r}^2$ and $\delta_{r,d}^2$, respectively.

At the destination, it combines the received signals $\mathbf{y}_{s,d}$ and $\mathbf{y}_{r,d}$ from both phases to jointly detect the original message \mathbf{x}_s . The combined signal at the destination can be expressed as

$$\mathbf{y}_d \triangleq \begin{pmatrix} \mathbf{y}_{s,d} \\ \mathbf{y}_{r,d} \end{pmatrix} = \begin{pmatrix} \sqrt{P_1} \mathbf{H}_{s,d} \mathbf{x}_s \\ \sqrt{\tilde{P}_2} \mathbf{H}_{r,d} \mathbf{x}_r \end{pmatrix} + \begin{pmatrix} \mathbf{n}_{s,d} \\ \mathbf{n}_{r,d} \end{pmatrix}. \quad (158)$$

Since $\mathbf{x}_r = \mathbf{G}\mathbf{x}_s$, the combined signal \mathbf{y}_d can be further simplified as

$$\mathbf{y}_d = \mathbf{H}_d \mathbf{x}_s + \mathbf{n}_d, \quad (159)$$

where $\mathbf{H}_d \triangleq \begin{pmatrix} \sqrt{P_1} \mathbf{H}_{s,d} \\ \sqrt{\tilde{P}_2} \mathbf{H}_{r,d} \mathbf{G} \end{pmatrix}$, $\mathbf{n}_d \triangleq \begin{pmatrix} \mathbf{n}_{s,d} \\ \mathbf{n}_{r,d} \end{pmatrix}$ and $\mathbf{n}_d \sim \mathcal{CN}(0, \mathcal{N}_0 \mathbf{I}_{L+K})$.

3.2.5 Optimum Linear Mapping Design

In this subsection, we optimize the linear mapping matrix \mathbf{G} of size $K \times L$ ($K < L$) such that the outage probability of the proposed linear-mapping based cooperative protocol is minimized.

First, we derive the outage probability of the linear-mapping based cooperative protocol. From (155), we observe that the channel from the source to the destination in Phase I is a multi-input multi-output complex

Gaussian channel. Thus, with i.i.d. circular symmetric complex Gaussian inputs, the maximum mutual information between the source and the destination in Phase I is

$$\begin{aligned}\mathcal{I}_{s,d} &= \frac{1}{L+K} \log_2 \left[\det \left(\mathbf{I}_L + \frac{P_1}{\mathcal{N}_0} \mathbf{H}_{s,d} \mathbf{H}_{s,d}^H \right) \right] \\ &= \frac{L}{L+K} \log_2 \left(1 + \frac{P_1}{\mathcal{N}_0} |h_{s,d}|^2 \right),\end{aligned}\quad (160)$$

where the factor $\frac{1}{L+K}$ in the first equality is due to the fact that the cooperative protocol uses $L+K$ time slots in Phase I. Similarly, the maximum mutual information between the source and the relay in Phase I is given by

$$\mathcal{I}_{s,r} = \frac{L}{L+K} \log_2 \left(1 + \frac{P_1}{\mathcal{N}_0} |h_{s,r}|^2 \right). \quad (161)$$

In Phase II, if the relay decodes the message from the source correctly (i.e., $\tilde{P}_2 = P_2$), the destination can utilize the combined signal \mathbf{y}_d from two phases to detect the original message \mathbf{x}_s . From (159), we can see that the channel between the output \mathbf{y}_d and the input \mathbf{x}_s is an equivalent multi-input, multi-output complex Gaussian channel. Therefore, the maximum mutual information of the joint detection in Phase II is

$$\begin{aligned}\mathcal{I}_d &= \frac{1}{L+K} \log_2 \left[\det \left(\mathbf{I}_{L+K} + \frac{\mathbf{H}_d \mathbf{H}_d^H}{\mathcal{N}_0} \right) \right] \\ &= \frac{1}{L+K} \log_2 \left\{ \det \left[\left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} \right) \mathbf{I}_L + \frac{P_2 |h_{r,d}|^2}{\mathcal{N}_0} \mathbf{G}^H \mathbf{G} \right] \right\},\end{aligned}\quad (162)$$

where the second equality is due to the fact that $\det \left(\mathbf{I}_{L+K} + \frac{\mathbf{H}_d \mathbf{H}_d^H}{\mathcal{N}_0} \right) = \det \left(\mathbf{I}_L + \frac{\mathbf{H}_d^H \mathbf{H}_d}{\mathcal{N}_0} \right)$ and $\mathbf{H}_d^H \mathbf{H}_d = \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} \mathbf{I}_L + \frac{P_2 |h_{r,d}|^2}{\mathcal{N}_0} \mathbf{G}^H \mathbf{G}$. Thus, for any given target transmission rate R_T , the outage probability of the proposed linear-mapping based cooperative protocol is

$$\mathcal{P}_{out} = Pr [\mathcal{I}_{s,d} < R_T] Pr [\mathcal{I}_{s,r} < R_T] + (1 - Pr [\mathcal{I}_{s,r} < R_T]) Pr [\mathcal{I}_d < R_T], \quad (163)$$

in which $\mathcal{I}_{s,d}$, $\mathcal{I}_{s,r}$ and \mathcal{I}_d are specified in (160), (161) and (162), respectively.

Next, we try to optimize the linear mapping \mathbf{G} in order to minimize the outage probability in (163). We note that in (163), only the term \mathcal{I}_d depends on the linear mapping \mathbf{G} . Thus, to minimize the outage probability \mathcal{P}_{out} , it is equivalent to minimize $Pr [\mathcal{I}_d < R_T]$ under the power constraint $Tr (\mathbf{G} \mathbf{G}^H) \leq K$. The power constraint ensures that the forwarded signals have unit average power. The mutual information \mathcal{I}_d in (162) can be further calculated as

$$\begin{aligned}\mathcal{I}_d &= \frac{1}{L+K} \log_2 \left[\left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} \right)^L \det \left(\mathbf{I}_K + \frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0} \mathbf{G} \mathbf{G}^H \right) \right] \\ &\triangleq \frac{1}{L+K} (\mathcal{I}_{d,1} + \mathcal{I}_{d,2}),\end{aligned}\quad (164)$$

where $\mathcal{I}_{d,1} \triangleq \log_2 \left(1 + \frac{P_1 |h_{s,d}|^2}{\mathcal{N}_0} \right)^L$ and $\mathcal{I}_{d,2} \triangleq \log_2 \left[\det \left(\mathbf{I}_K + \frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0} \mathbf{G} \mathbf{G}^H \right) \right]$. We observe that $\mathcal{I}_{d,1}$ does not depend on the linear mapping \mathbf{G} . Thus, to maximize \mathcal{I}_d , it is equivalent to maximize $\mathcal{I}_{d,2}$ under the power constraint that $Tr (\mathbf{G} \mathbf{G}^H) \leq K$. To further calculate $\mathcal{I}_{d,2}$, we consider eigen-decomposition $\mathbf{G} \mathbf{G}^H = \mathbf{U}^H \mathbf{\Lambda} \mathbf{U}$, where \mathbf{U} is a unitary matrix, $\mathbf{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_K)$ and the eigenvalues $\lambda_1, \dots, \lambda_K$ are non-negative. Thus, we have

$$\begin{aligned}\mathcal{I}_{d,2} &= \log_2 \left[\det \left(\mathbf{I}_K + \frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0} \mathbf{U}^H \mathbf{\Lambda} \mathbf{U} \right) \right] \\ &= \sum_{i=1}^K \log_2 \left(1 + \frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0} \lambda_i \right).\end{aligned}\quad (165)$$

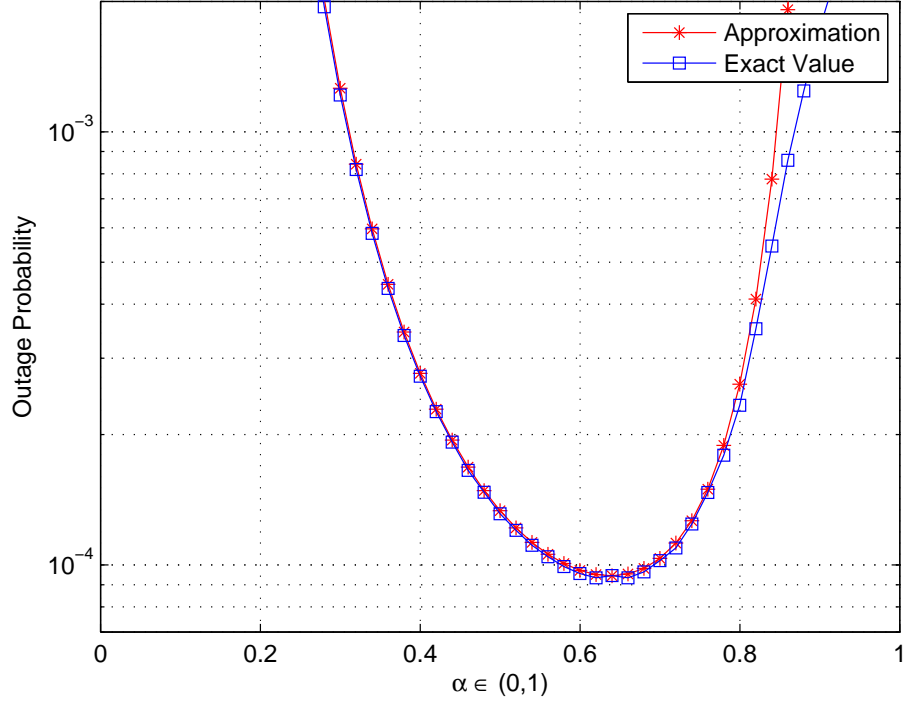


Figure 15: Outage probability of the ideal cooperative protocol with different time allocation ratio $\alpha \in (0, 1)$ under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1\}$. Assume that $P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ for any given $\alpha \in (0, 1)$ based on Theorem 5, and $P/\mathcal{N}_0 = 30\text{dB}$.

Since $\sum_{i=1}^K \lambda_i = \text{Tr}(\mathbf{G}\mathbf{G}^H) \leq K$, the problem of optimizing the linear mapping G is specified as

$$\begin{aligned} \max_{\lambda_1, \dots, \lambda_K} \quad & \sum_{i=1}^K \log_2 \left(1 + \frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0} \lambda_i \right) \\ \text{s.t.} \quad & \sum_{i=1}^K \lambda_i \leq K. \end{aligned} \quad (166)$$

We can solve the problem by using Lagrange multipliers. Consider an Lagrange multiplier μ and an Lagrange function

$$\mathcal{L}(\lambda_1, \dots, \lambda_K) = \sum_{i=1}^K \log_2 \left(1 + \frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0} \lambda_i \right) + \mu \left(\sum_{i=1}^K \lambda_i \right). \quad (167)$$

Differentiating the Lagrange function with respect to λ_i , we have

$$\frac{\partial \mathcal{L}(\lambda_1, \dots, \lambda_K)}{\partial \lambda_i} = \frac{1}{\lambda_i + \frac{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0}{P_2 |h_{r,d}|^2 / \mathcal{N}_0}} + \mu. \quad (168)$$

Let $\frac{\partial \mathcal{L}(\lambda_1, \dots, \lambda_K)}{\partial \lambda_i} = 0$, we obtain an expression of optimum λ_i as

$$\lambda_i = \max(0, -\frac{1}{\mu} - \frac{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0}{P_2 |h_{r,d}|^2 / \mathcal{N}_0}). \quad (169)$$

Since $\sum_{i=1}^K \lambda_i = K$, we have

$$\mu = -\frac{P_2 |h_{r,d}|^2 / \mathcal{N}_0}{1 + P_1 |h_{s,d}|^2 / \mathcal{N}_0 + P_2 |h_{r,d}|^2 / \mathcal{N}_0}. \quad (170)$$

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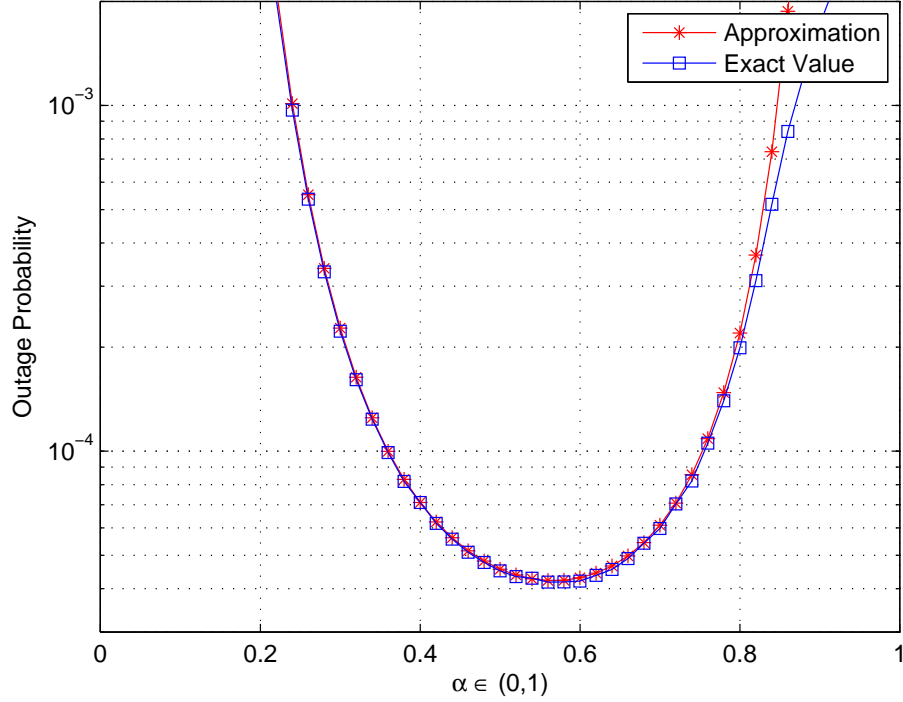


Figure 16: Outage probability of the ideal cooperative protocol with different time allocation ratio $\alpha \in (0, 1)$ under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{10, 1\}$. Assume that $P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ for any given $\alpha \in (0, 1)$ based on Theorem 5, and $P/\mathcal{N}_0 = 30\text{dB}$.

By substituting (170) into (169), we have

$$\lambda_i = \frac{1 + P_1|h_{s,d}|^2/\mathcal{N}_0 + P_2|h_{r,d}|^2/\mathcal{N}_0}{P_2|h_{r,d}|^2/\mathcal{N}_0} - \frac{1 + P_1|h_{s,d}|^2/\mathcal{N}_0}{P_2|h_{r,d}|^2/\mathcal{N}_0} = 1, \quad (171)$$

i.e., the optimum solution for the problem (166) is $\lambda_i = 1$ for $i = 1, \dots, K$. Therefore, the linear mapping \mathbf{G} minimizes the outage probability in (163) if and only if $\mathbf{G}\mathbf{G}^H = \mathbf{I}_K$. We summarize the above discussion in the following theorem.

Theorem 7 *In the linear-mapping based cooperative protocol, any $K \times L$ ($K < L$) linear mapping \mathbf{G} with power constraint $\text{Tr}(\mathbf{G}\mathbf{G}^H) \leq K$ minimizes the outage probability of the protocol if and only if it satisfies $\mathbf{G}\mathbf{G}^H = \mathbf{I}_K$. Consequently, we may select any K rows of an $L \times L$ unitary matrix to form the optimum linear mapping \mathbf{G} .*

3.2.6 Numerical and Simulation Results

In this section, we provide some numerical studies and simulation results to illustrate the performance benchmark of the ideal cooperative protocol and the performance of the practical cooperative protocol based on the optimum linear mapping. In all numerical studies and simulations, we assume that the target transmission rate is $R_T = 2$ bits/s/Hz, the noise variance is $\mathcal{N}_0 = 1$, and the variance of the source-destination channel is normalized as $\delta_{s,d}^2 = 1$. We consider three exemplary scenarios with various channel variances: (i) $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1\}$; (ii) $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{10, 1\}$; and (iii) $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 10\}$.

In Figs. 15–17, we plot the outage probability of the ideal cooperative protocol in terms of different time allocation ratio $\alpha \in (0, 1)$ for the three channel conditions (i)–(iii), respectively. In the study, we assume $P/\mathcal{N}_0 = 30\text{dB}$. For any given time allocation ratio $\alpha \in (0, 1)$, the corresponding optimum power allocation

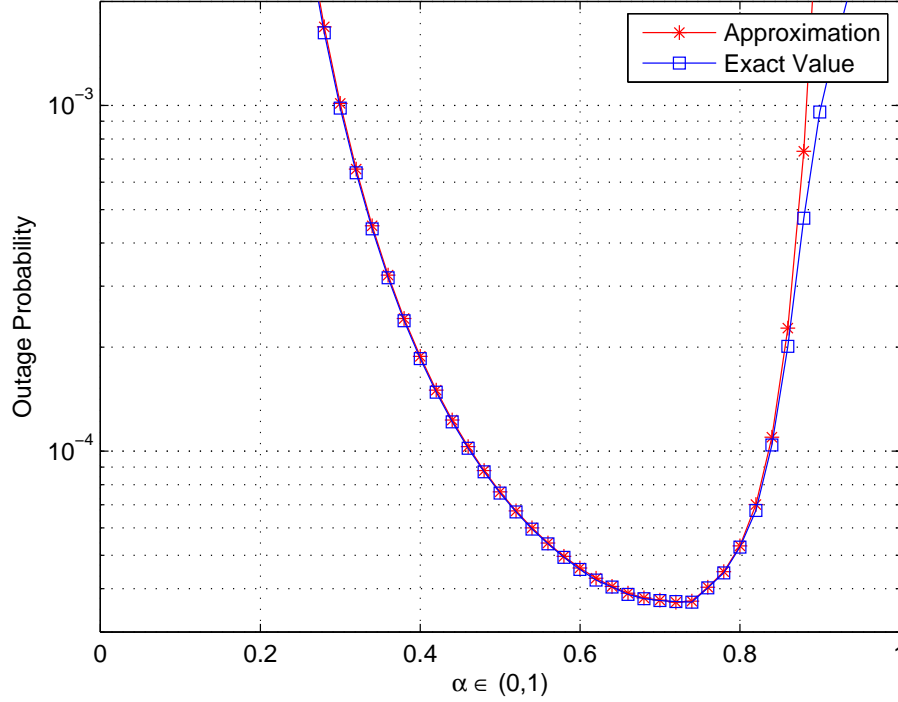


Figure 17: Outage probability of the ideal cooperative protocol with different time allocation ratio $\alpha \in (0, 1)$ under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 10\}$. Assume that $P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ for any given $\alpha \in (0, 1)$ based on Theorem 5, and $P/\mathcal{N}_0 = 30\text{dB}$.

$P_1 = P_1^*(\alpha)$ and $P_2 = P_2^*(\alpha)$ are calculated based on Theorem 5. In each figure, we plot the outage probability approximation based on (133) as well as the exact outage probability calculated based on (121) for comparison. In the case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, Fig. 15 shows that the optimum time allocation ratio is $\alpha^* = 0.66$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.0971P$ and $P_2^*(\alpha^*) = 0.8115P$ based on Theorem 5, in which the energy allocation ratio $\beta = 0.7192$. In case of $\delta_{s,r}^2 = 10$ and $\delta_{r,d}^2 = 1$, Fig. 16 shows that the optimum time allocation ratio is $\alpha^* = 0.59$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.0196P$ and $P_2^*(\alpha^*) = 0.9718P$ based on Theorem 5 and the energy allocation ratio $\beta = 0.5969$. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, Fig. 17 shows that the optimum time allocation ratio is $\alpha^* = 0.74$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.1112P$ and $P_2^*(\alpha^*) = 0.6835P$ and the energy allocation ratio $\beta = 0.8122$. Figs. 15 - 17 show that for all three channel conditions, the optimum time allocation ratio α^* is strictly larger than $\frac{1}{2}$ which is consistent to the theoretical development in Theorem 6, and the corresponding energy allocation ratio β is strictly larger than $\frac{1}{2}$ which is consistent to the result in Theorem 5. Moreover, we observe that the larger the ratio of the relay-destination link quality over the source-relay link quality ($\delta_{r,d}^2/\delta_{s,r}^2$) is, the larger the optimum time allocation ratio α^* and the corresponding energy allocation ratio β are. In the three figures, we can see that the approximation of the outage probability based on (133) matches the exact value of the outage probability based on (121) very well.

Figs. 18–20 show the outage probability performance of the proposed practical cooperative relaying protocol based on the optimum linear mapping for three channel conditions (i)–(iii), respectively. For comparison, the figures also show the performance of the direct transmissions, the performance of the cooperative protocol based on equal time allocation, and the performance benchmark from the ideal cooperative protocol. Table 4 specifies time and power parameters (α, P_1, P_2) for each cooperative protocol under the three channel conditions, respectively. When $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, Fig. 18 shows that the proposed practical cooperative protocol outperforms the equal-time based cooperative protocol with performance improvement about 1.5dB, and the difference between the performances of the practical linear-mapping based cooperative protocol and the ideal

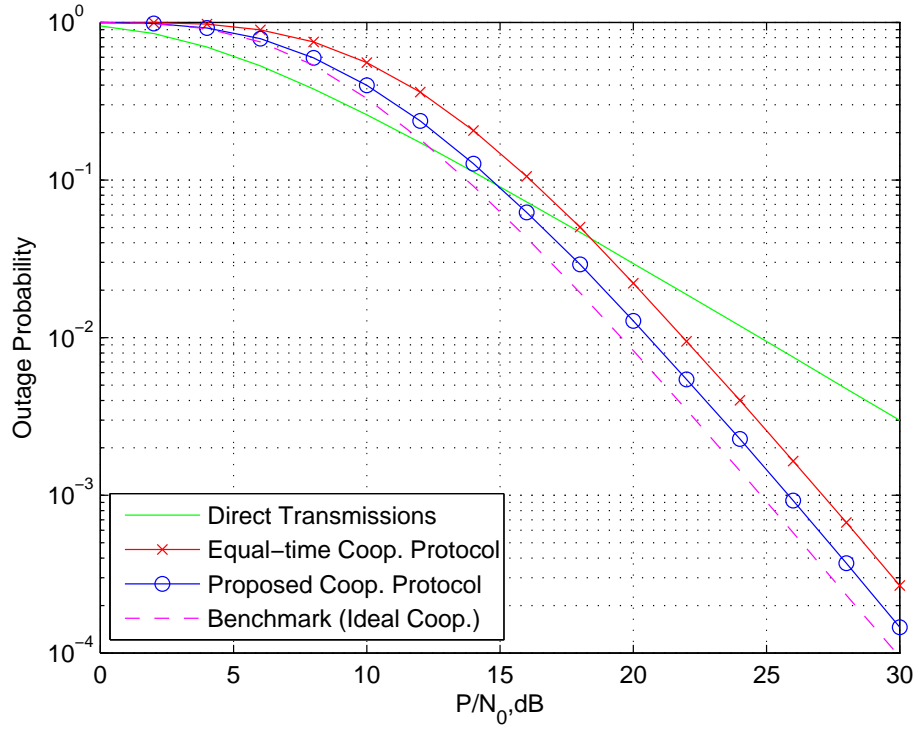


Figure 18: Performance of the proposed practical cooperative protocol with the optimum linear mapping under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1\}$.

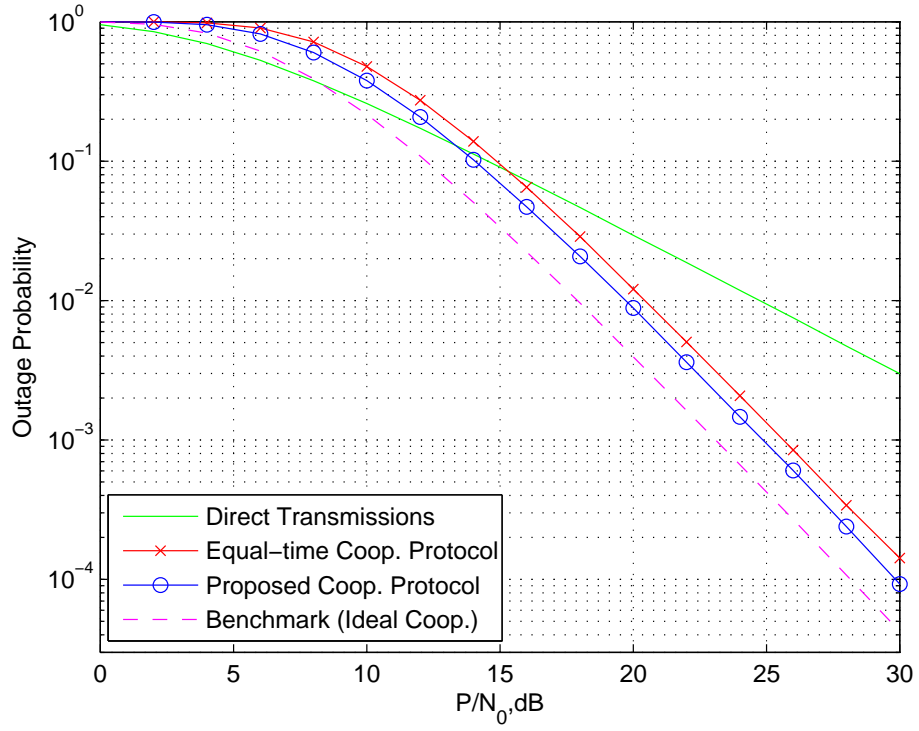


Figure 19: Performance of the proposed practical cooperative protocol with the optimum linear mapping under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{10, 1\}$.

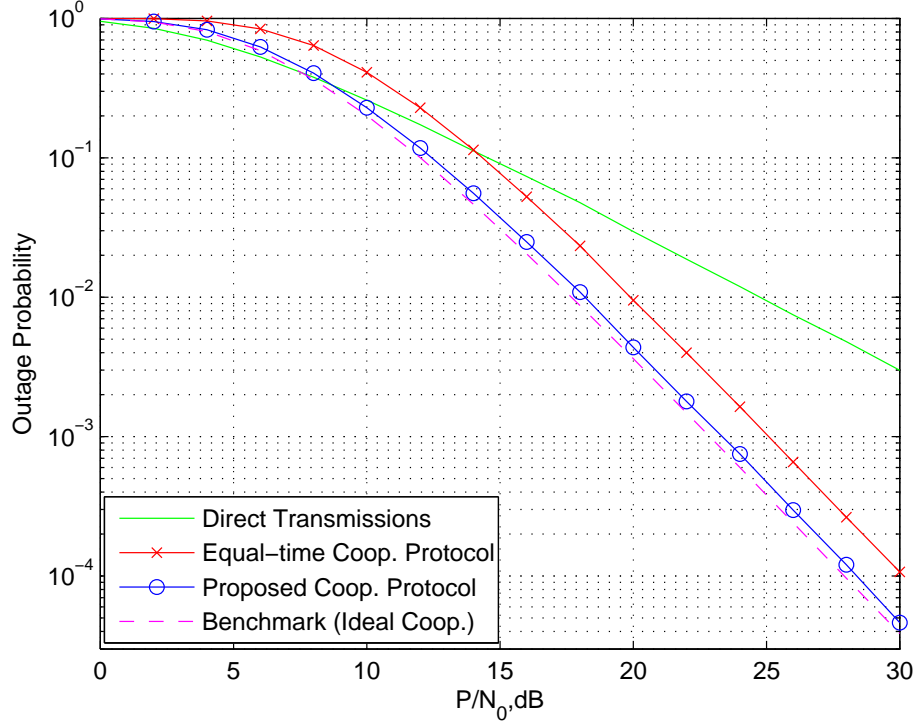


Figure 20: Performance of the proposed practical cooperative protocol with the optimum linear mapping under the channel condition $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 10\}$.

benchmark is less than 1dB. When $\delta_{s,r}^2 = 10$ and $\delta_{r,d}^2 = 1$, we can see in Fig. 19 that there is a 2dB difference between the proposed practical cooperative protocol and the ideal performance benchmark, but compared to the equal-time based cooperative protocol, the performance difference is less than 1dB. In this case, the performance gain over the equal-time based cooperative protocol is narrowed because the equal-time allocation ratio is close to the optimum time ratio which is $\alpha^* = 0.59$ in this case. When $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, Fig. 20 shows that the performance of the proposed practical cooperative protocol has over 2.5dB improvement compared to the equal-time based cooperative protocol. In this case, the performance of the proposed linear-mapping based cooperative protocol is very close to the ideal performance benchmark (less than 0.25dB). From the figures, we can see that the equal-time based cooperative protocol is not optimum in general, and the proposed practical cooperative protocol with the optimum linear mapping has performance gain in all three different channel conditions, which varies according to channel conditions. Moreover, we observe that when the ratio of the relay-destination link quality over the source-relay link quality ($\delta_{r,d}^2/\delta_{s,r}^2$) becomes larger, the gap between the performance of the proposed practical linear-mapping based cooperative protocol and the ideal benchmark becomes smaller.

Table 4: TIME AND POWER PARAMETERS FOR DIFFERENT CHANNEL CONDITIONS

$\{\delta_{s,r}^2, \delta_{r,d}^2\}$	Resource Allocation (α, P_1, P_2)		
	Ideal Coop.	Proposed Coop.	Equal-time Coop.
$\{1, 1\}$	$(0.66, 1.0971P, 0.8115P)$	$(\frac{2}{3}, 1.0788P, 0.8423P)$	$(\frac{1}{2}, 1.6328P, 0.3672P)$
$\{10, 1\}$	$(0.59, 1.0196P, 0.9718P)$	$(\frac{3}{5}, 0.9948P, 1.0079P)$	$(\frac{1}{2}, 1.2946P, 0.7054P)$
$\{1, 10\}$	$(0.74, 1.1112P, 0.6835P)$	$(\frac{3}{4}, 1.0829P, 0.7513P)$	$(\frac{1}{2}, 1.8560P, 0.1440P)$

4 Results and Discussions

In this project, we attacked the fundamental problem of assigning optimal transmission power sequence for cooperative H-ARQ relaying protocol over quasi-static Rayleigh fading channels. Finally, we are able to determine an optimal transmission power assignment strategy for the cooperative H-ARQ relaying protocol over quasi-static Rayleigh fading channels to minimize the average total power consumption. We investigated for the first time the average total transmission power consumed by the cooperative H-ARQ protocol. Specifically, we developed an analytical approach to obtain a closed-form expression of the average total transmission power which is valid for any maximum number of (re)transmission rounds L allowed in the protocol. However, the closed-form expression of the average total transmission power is complicated in general, so we developed a simple approximation of the average total transmission power which is tight at high SNR. Then, based on the asymptotically tight approximation, we determined the optimal power sequence that minimizes the average total power consumption of the protocol for any given targeted outage probability. We derived a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. The optimal power assignment solution reveals that conventional equal power assignment scheme is far from optimal. Numerical and simulation results illustrate and validate our theoretical development. Specifically, in the first set of numerical studies, we compared the average total transmission power of the cooperative H-ARQ relaying protocol by comparing the approximation result in (75), the exact closed-form result in (26) and the simulation result for the cases of $L = 2, 3$ and 4 , respectively. By comparisons, we observed that the closed-form result of the average total transmission power matches exactly with the simulation curve in each study. We also observed that the approximation of the average total transmission power is loose at low SNR and tight at high SNR. The approximation curve is almost indistinguishable from the exact closed-form result and the simulation curve for SNR above 15 dB in each study. In the second set of numerical studies, we compared the optimal transmission power sequence from the Theorem and the exhaustive search result based on the original optimization problem in (82) (without approximation). We observed that the optimal transmission power values resulted from the Theorem match well with the exhaustive search result from the original optimization problem. An interesting observation is that the optimal power assignment strategy assigns less transmission power to the source in the first few (re)transmission rounds and significantly large transmission power to the source at the last (L -th) round compared to the equal power assignment strategy. It provides some insightful information on how much power should be assigned to the source at each round for saving the average total power cost. In the third set of numerical studies, we compared the average total transmission power for both the optimal power assignment scheme and the equal power assignment scheme, with different targeted SNR γ_0 (from 0 dB to 25 dB). The required outage performance is set at $p_0 = 10^{-3}$. When $L = 2$, we observed that the optimal power assignment saves about 1.5 dB in average total transmission power compared to the equal power assignment. When $L = 3$ and 4 , we observed that the optimal power assignment scheme outperforms the equal power assignment scheme with a performance improvement of about 2.6 dB. Moreover, it is interesting to observe that the performance gain of the optimal power assignment scheme is almost constant for different targeted SNR γ_0 (from 0 dB to 25 dB). In the fourth set of numerical studies, we compared the average total transmission power required in the two power assignment strategies with different targeted outage probability values for the cases of $L = 2, 3$ and 4 , respectively. From the comparisons, we observed that for an outage performance of $p_0 = 10^{-5}$, the power savings of the optimal power assignment compared to the equal power assignment are 5 dB when $L = 2$, 10 dB when $L = 3$, and 11 dB when $L = 4$. We also observed that the lower the required outage probability, the more performance gain of the optimal power assignment strategy compared to the equal power assignment strategy.

While most existing works on cooperative relaying protocol designs considered equal-time allocation scenario, i.e. equal time duration is assigned to each source and each relay, in this project we are able to design and optimize cooperative communication protocols by exploring all possible variations in time and power domains. First, we analyzed and optimized the ideal cooperative communication protocol where the system can use arbitrary re-encoding function at the relay and adjust time allocation arbitrarily between Phases I and II. Based on the asymptotically tight approximation of the outage probability, we obtained the optimum strategy of power and time allocations to minimize the outage probability of the ideal cooperative protocol. For any given time

allocation $\alpha \in (0, 1)$, we determined the corresponding optimum power allocation at the source and the relay analytically with a closed-form expression. We also showed theoretically that in order to minimize the outage probability of the protocol, one should always allocate more energy and time to Phase I than that to Phase II in the protocol. We note that in the ideal cooperative protocol in which there is no constraint on the re-encoding methods and time allocation, it may not be easy/feasible to implement it in practical systems. Therefore, with more realistic consideration, we proposed a practical cooperative relaying protocol design based on linear mapping, where the protocol considers linear mapping forwarding method at the relay and uses integer time slots in Phases I and II. The theoretical results from the ideal cooperative protocol served as guideline and benchmark in the practical cooperative protocol design. We also developed an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Extensive numerical and simulation results show that the practical cooperative relaying protocol based on the optimum linear mapping outperforms the existing cooperative protocol with equal time allocation, and more interestingly, the performance of the practical linear-mapping based cooperative relaying protocol is close to the performance benchmark of the ideal cooperative protocol. More specifically, in numerical studies and simulations, we consider three exemplary scenarios with various channel variances: (i) $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 1\}$; (ii) $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{10, 1\}$; and (iii) $\{\delta_{s,r}^2, \delta_{r,d}^2\} = \{1, 10\}$. The variance of the source-destination channel is normalized as $\delta_{s,d}^2 = 1$. In one set of numerical studies, we compared the outage probability of the ideal cooperative protocol in terms of different time allocation ratio $\alpha \in (0, 1)$ for the three channel conditions (i)-(iii), respectively. In the study, we compared the outage probability approximation based on (133) as well as the exact outage probability calculated based on (121). In the case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, it shows that the optimum time allocation ratio is $\alpha^* = 0.66$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.0971P$ and $P_2^*(\alpha^*) = 0.8115P$ based on Theorem 5, in which the energy allocation ratio $\beta = 0.7192$. In case of $\delta_{s,r}^2 = 10$ and $\delta_{r,d}^2 = 1$, it shows that the optimum time allocation ratio is $\alpha^* = 0.59$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.0196P$ and $P_2^*(\alpha^*) = 0.9718P$ based on Theorem 5 and the energy allocation ratio $\beta = 0.5969$. In case of $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, it shows that the optimum time allocation ratio is $\alpha^* = 0.74$. The corresponding optimum power allocation is $P_1^*(\alpha^*) = 1.1112P$ and $P_2^*(\alpha^*) = 0.6835P$ and the energy allocation ratio $\beta = 0.8122$. We observed that for all three channel conditions, the optimum time allocation ratio α^* is strictly larger than $\frac{1}{2}$ which is consistent to the theoretical development in Theorem 6, and the corresponding energy allocation ratio β is strictly larger than $\frac{1}{2}$ which is consistent to the result in Theorem 5. Moreover, we observed that the larger the ratio of the relay-destination link quality over the source-relay link quality ($\delta_{r,d}^2/\delta_{s,r}^2$) is, the larger the optimum time allocation ratio α^* and the corresponding energy allocation ratio β are. In another set of numerical studies, we observed the outage probability performance of the proposed practical cooperative relaying protocol based on the optimum linear mapping for three channel conditions (i)-(iii), respectively. For comparison, we also simulated the performance of the direct transmissions, the performance of the cooperative protocol based on equal time allocation, and the performance benchmark from the ideal cooperative protocol. In comparisons, we observed that when $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 1$, the proposed practical cooperative protocol outperforms the equal-time based cooperative protocol with performance improvement about 1.5dB, and the difference between the performances of the practical linear-mapping based cooperative protocol and the ideal benchmark is less than 1dB. When $\delta_{s,r}^2 = 10$ and $\delta_{r,d}^2 = 1$, we observed that there is a 2dB difference between the proposed practical cooperative protocol and the ideal performance benchmark, but compared to the equal-time based cooperative protocol, the performance difference is less than 1dB. In this case, the performance gain over the equal-time based cooperative protocol is narrowed because the equal-time allocation ratio is close to the optimum time ratio which is $\alpha^* = 0.59$ in this case. When $\delta_{s,r}^2 = 1$ and $\delta_{r,d}^2 = 10$, we observed that the performance of the proposed practical cooperative protocol has over 2.5dB improvement compared to the equal-time based cooperative protocol. In this case, the performance of the proposed linear-mapping based cooperative protocol is very close to the ideal performance benchmark (less than 0.25dB). From the comparisons, we can conclude that the equal-time based cooperative protocol is not optimum in general, and the proposed practical cooperative protocol with the optimum linear mapping has performance gain in all three different channel conditions, which varies according to channel conditions.

5 Conclusions

In this report, we summarized our findings resulting from the project and described in details the system models and the proposed methods and procedures. First, we determined an optimal transmission power assignment strategy for the cooperative H-ARQ relaying protocol over quasi-static Rayleigh fading channels to minimize the average total power consumption. We investigated for the first time the average total transmission power consumed by the cooperative H-ARQ protocol. Specifically, we developed an analytical approach to obtain a closed-form expression of the average total transmission power which is valid for any maximum number of (re)transmission rounds L allowed in the protocol. However, the closed-form expression of the average total transmission power is complicated in general, so we developed a simple approximation of the average total transmission power which is tight at high SNR. Then, based on the asymptotically tight approximation, we determined the optimal power sequence that minimizes the average total power consumption of the protocol for any given targeted outage probability. We derived a set of equations that describe the optimal power level in each (re)transmission and enable its recursive calculation with fixed searching complexity. The optimal power assignment solution reveals that conventional equal power assignment scheme is far from optimal. For example, for a targeted outage performance of $p_0 = 10^{-3}$ and maximum number of (re)transmissions $L = 4$, the optimal power assignment scheme saves about 2.6 dB in the average total power consumption compared to the equal power assignment scheme. We also observe that the lower the required outage probability, the more performance gain of the optimal power assignment scheme comparing to the equal power scheme.

Furthermore, we designed and optimized cooperative relaying protocols by exploring possible variations in time and power domains. First, we analyzed and optimized the ideal cooperative communication protocol where the system can use arbitrary re-encoding function $\mathcal{M}(\cdot)$ at the relay and adjust time allocation arbitrarily between Phases I and II. Based on the asymptotically tight approximation of the outage probability, we obtained the optimum strategy of power and time allocations to minimize the outage probability of the ideal cooperative protocol. For any given time allocation $\alpha \in (0, 1)$, we determined the corresponding optimum power allocation at the source and the relay analytically with a closed-form expression. We also showed theoretically that in order to minimize the outage probability of the protocol, one should always allocate more energy and time to Phase I than that to Phase II in the protocol. We note that in the ideal cooperative protocol in which there is no constraint on the re-encoding methods and time allocation, it may not be easy/feasible to implement it in practical systems. Therefore, with more realistic consideration, we proposed a practical cooperative relaying protocol design based on linear mapping, where the protocol considers linear mapping forwarding method at the relay and uses integer time slots in Phases I and II. The theoretical results from the ideal cooperative protocol served as guideline and benchmark in the practical cooperative protocol design. We also developed an optimum linear mapping to minimize the outage probability of the linear-mapping based cooperative protocol. Simulation results show that the practical cooperative relaying protocol based on the optimum linear mapping outperforms the existing cooperative protocol with equal time allocation, and more interestingly, the performance of the practical linear-mapping based cooperative relaying protocol is close to the performance benchmark of the ideal cooperative protocol. We observed that when the ratio of the relay-destination link quality over the source-relay link quality ($\delta_{r,d}^2/\delta_{s,r}^2$) becomes larger, the gap between the performance of the proposed linear-mapping based cooperative protocol and the ideal benchmark becomes smaller.

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ACRONYMS

ACK	ACKnowledgement
AF	Amplify-and-Forward
ARQ	Automatic-Repeat-reQuest
AWGN	Additive White Gaussian Noise
CDF	Cumulative Distribution Function
CSI	Channel State Information
DF	Decode-and-Forward
H-ARQ	Hybrid Automatic-Repeat-reQuest
LTE	Long Term Evolution (LTE)-Advanced standard
MIMO	Multiple-Input-Multiple-Output
MRC	Maximal-Ratio-Combining
NACK	Negative-ACKnowledgement
QoS	Quality-of-Service
SER	symbol error rate
SNR	Signal-to-Noise Ratio